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Al-Bīrūnī on Transits

A Study of an Arabic Treatise entitled
Tamhīd al-mustaqarr li-taḥqīq ma'nā al-mamarr
by
Abū l-Rayḥān al-Bīrūnī
(d. 440/1048)

Translated by Mohammad Saffouri & Adnan Ifram
with a commentary by Edward S. Kennedy

including a review by
G.J. Toomer

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AI-BĪRŪNĪ on TRANSITS

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Al-BĪRŪNĪ on TRANSITS

A Study of an Arabic Treatise
entitled

تمهيد المستقر لتحقيق معنى الممر

by

Abū al-Rayḥān,
Muḥammad ibn Aḥmad al-Bīrūnī
(d. 1048)

translated by
Mohammad Saffouri & Adnan Ifram
with a commentary by
E. S. Kennedy

1959

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Preface

This book makes available in English translation one of the minor works of an individual who was at once a versatile contributor to the science of his own day and a matchless critic and historian of the scientific lore of his predecessors. Bīrūnī was able to use many sources which have since disappeared, and his writings afford us part of the means for eventually tracing the transmission of astronomical theory between the Near East, India, and Iran.

The reader must not hope to find here a synthesis of Islamic astronomy. Our author set himself the task of examining the ramifications of a particular concept which is more astrological than astronomical. The reading of his results is not made easier by the fact that he felt himself constrained to write in terms of the best planetary theory of his day, that of Ptolemy, whereas the techniques he describes seem to have been worked out in the context of a more primitive body of theory. Nevertheless a study of the text leaves us with a reasonably adequate understanding of the main topic. But what is vastly more rewarding is the collection of byproducts. This treatise is a veritable mine of numerical parameters, in certain cases whole sets of related planetary constants which can be made secure by internal cross-checkings. There are a number of quotations from lost works, and all manner of incidental statements bearing usefully on a variety of topics.

It has been our effort to translate the entirety of the text as faithfully as we could, to explain in terms of modern symbols those sections which seemed to require explanation, to point out those which remain obscure to us, to recompute and verify numerical material where possible, and to make appropriate references to the literature. Of the shortcomings in the result we are all too aware.

The unique extant manuscript copy of the original text is Arabic Ms. 2468/38 of the Oriental Public Library

(Bankipore), Patna, India. Thus far, however, we have been unable to secure a microfilm of this manuscript, and the translation has been prepared from the published version of the text, the third of four treatises bound together under the title Rasā'ilul'Bīrūnī. It was printed in 1948 by the Osmania Oriental Publications Bureau, Hyderabad-Deccan, India, as one of a series of important texts being published by the Bureau.

A preliminary translation was made by Mr. Saffouri during the academic year 1956-57, under a grant from the American University of Beirut. Large sections of the text at this stage remained unintelligible to both of us. During the fall semester of 1957 I worked systematically through the original, improving the translation, and discussing partial results as they were obtained, in a seminar held at Brown University. Professors O. Neugebauer and A. Aaboe, who participated in the seminar, made many fruitful suggestions involving all aspects of the work. This phase of the job was continued during the succeeding spring term at the Institute for Advanced Study. Thus the substance of the text was in large measure made clear, and during the current academic year Mr. Ifram and I made a thorough revision of the translation. The latter was done on time made available by a grant from the National Science Foundation, Washington, D.C. Copy for the photo-offset reproduction was typed by Mrs. Kawthar A. Shomar. The title pages were designed by Professor John Carswell, and the Arabic title is in the hand of Mirza Nur-ud-Din Zeine.

To the institutions and individuals mentioned above we express deep gratitude, while retaining for ourselves the responsibility for all mistakes which this edition may contain.

E.S.K.

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TRANSLATION OF THE TEXT

For ease of reference, the translation is displayed according to the pages and lines of the published Arabic text. The numbers in the upper left-hand corner of each page of the translation give the page of the text and, following the colon, the number of the particular line with which that page begins. The column of numbers below gives only line numbers except where a new page of text begins. Readers referring back to the text (the <u>Rasā'il</u>) should note that in it each of the four treatises it contains is paginated separately. In general, parentheses in the translation enclose words or phrases not in the original, but added for clarification. Square brackets in the translation enclose restorations to the text. Except for restored letters on the figures, all such restorations are noted in the commentary, the Arabic both of the text and the emendation being given.

Paragraphs in the translation are those of the printed text.

TRANSLATION

1:1 In the name of God, the Merciful the Forgiving.

2 Abū al-Rayḥān; may God have mercy on him, said:
3 Transit (mamarr), in the language, is derived from crossing (ijtiyāz) meaning either the actual act (of crossing)
4 or the place where the doer (i.e. the crosser) may be. And so it may be interpreted as (either the act of) crossing or the
5 place of crossing; and to (either of) these two meanings the astrologers (al-munajjimūn) refer when they use it. Then
6 they give it a special meaning in their craft which they call exceptional to the laws of language.
7 The ether is a body having three dimensions of which the length (al-ṭūl) is by convention longer than the width (al-ʿarḍ).
8 But the great circle on the sphere is its longest regular distance.
9 Hence length (or longitude) for it is the (great) circle (manṭaqa) of its motion, and width (or latitude) is what crosses (muʿtariḍ)
10 the length. And hence in the sphere it is what is between its (i.e. the sphere's) great circle (of motion) and its
11 two poles. And thickness is by necessity what is between the two ends of the ether along the diameter of the sphere;
12 one of these two ends is the lower one, I mean the concavity of the moon's heaven. And the other
13 is the upper one, which is the convexity of the circle (or roundness) where what exists ends and where is the extinction of existence.
14 And transit (as) mentioned in astrology deals with each one of the three dimensions.

2:1 **Mention (<u>or</u> Explanation) of Longitudinal Transit (<u>or</u> Transit in Length)**

2 Since the primary simple motions in the heaven are
3 two, western and eastern, and the transit of the planets has little connection with the western (i.e. diurnal motion) of the two (motions),
4 hence no planet will pass another because of them. Instead, it is said that a planet passes, by virtue of the two (motions), over the
5 position of another planet; or it moves along its track, or it deviates from it
6 to its left or to its right. And if they reach together one of the two circles, that of the horizon or the meridian,
7 while they differ in declination from the celestial equator, it is said, with respect to the horizon, that they rise together
8 or set together, and it is said, with respect to the meridian, that they reach midheaven together. But
9 if their declinations are equal in magnitude and direction, the times of their risings
10 and settings and of reaching midheaven would differ in all positions except at conjunction, if
11 they are, in addition to all that we have mentioned, in conjunction. This conjunction entails their coincidence
12 by sight (i.e., occultation) and the eclipsing of the upper by the lower one; however, this is a condition that hardly ever happens and is rarely
13 found.
14 And if their two times differ, (i.e., they have equal declinations but are) in other than this eclipsing position, nothing can be
15 said regarding them except that one of them rises at the rising place of the other and sets at its setting place

2:16 and reaches midheaven at the other's place of reaching it.
17 But the degree (of the ecliptic) where the planet meets the meridian in
18 latitude is not its degree (i.e., its longitude) if it is not at one of the two solstitial points. But it (the former degree) is called the degree
19 of transit. And this name is not used for the western motion except according to what we have mentioned.

3:1 And with reference to equality of azimuths, it is said that the transit of a certain star through a certain spot occurs
2 if its (the spot's) distance from the celestial equator equals its (the star's) distance. So its (the star's) equality (of azimuth to that of the spot) by this motion occurs once per
3 day, approximately.
4 By this westward motion the matter of the motion of the stars
5 and other (bodies) is explained as being what the stars and rays and so on are required (<u>or</u> fated) to move to.
6 And the meaning of <u>tasyīr</u> (is) that the planets which are made to move must be
7 at the assumed time, (either) on one of the two horizon circles or the meridian, or on a
8 circle between them which is one of the great circles which are horizons of places less in
9 latitude than the latitude of that horizon, passing through the intersection of this horizon and the
10 meridian. And if the sphere of the universe turns by the westward motion until (a planet) which is to be moved
11 reaches that circle on which was the first (planet) to be moved, then the degree

3:12 of the tasyīr is the time (measured in degrees along the celestial equator) which passes through that circle between the two (above-)mentioned cases (i.e., the two planets).

13 And the name of transit does not apply to it, even if one of the two planets passes over

14 the position of the other. And of the type of the longitudinal transit are the correspondences and disagreements of (zodiacal) signs.

15 They are mentioned in the Introductions (al-Mudākhil, to astrology) and the Bizīdhajāt (the Vizhīdhaks), and especially in the Rūmī (i.e., Byzantine) ones

16 where the meaning is implied by our terms, but if the words differ (from ours) it is due to our not having the book.

17 And that is that the signs correspond or differ (in course) according to their discrepancy in time (of daylight).

18 Thus some of them correspond in the arc of daylight if the numbers of equal hours in

19 their two days are equal, such as Gemini and Cancer, and as Taurus and Leo. And all such pairs of signs,

4:1 in general, are equally distant from a certain solstice. And their two days and the days

2 of all pairs of degrees of them, that are equally distant from the same solstice, are equal.

3 And just as their two days are equated so also are their doubles(?), and their ortive amplitudes,

4 and the noon altitude (of the sun when it is) at such pairs of points and the two shadows at them are in one direction, together with all that results from

5 the coincidence of the two small circles (madār). And the signs and degrees according to this meaning are paired. And each

6 one of every pair in the descending half (of the zodiac) which is from the beginning of Cancer

4:7 to the end of Sagittarius is commanding, and the one in the other, the ascending half (is called) obedient, and that is

8 by reference to the westward motion. Because if they rotate by it through one transit,

9 the one in advance would be the leader, and the other would be led. However, as to their two situations caused by the (variation in) inclination, straightening up

10 of their risings, and the increase in their oblique ascendings over their right ascensions,

11 and the obedience of the obedient (being) due to deformation of their risings and the decrease of their ascensions, that has been said (by others).

12 And the author of the Bizīdhaj called this type of signs corresponding in

13 strength (equipollent), as if he meant by strength the westward motion. And he said in another place

14 that the planet which is in Aries looks at that which is in Cancer,

15 and so it is its leader by the motion of the whole. And the one in Cancer accepts its radiation (i.e., that of the one in Aries) and follows it.

16 And he assigned the higher position to the western motion with the two small circles in agreement, and contented himself with aspect (naẓar).

17 And some signs agree in ascensions if these are equal for the locality, (i.e., in oblique ascension)

18 such as Aries and Pisces. And for each pair of signs equally distant from one and the same equinox

19 the times of their risings and the risings of all pairs of degrees fulfilling this condition,

5:1 are equal.

2 And Ptolemy calls the northern one elevated and the southern one, depressed. And it may be

5:3 that some of them (meaning the astrologers) call the
elevated one commandant and the depressed one obedient.
And as they are equal in ascensions,
4 so they are also equal in declination, and ortive
amplitudes, but in two different directions,
5 and their days (are) also equal, and all that results
from the equality of the two small circles.
6 And the author of *al-Bizīdhaj* has called this type the
ones that agree in ascensions. And he then
7 mentioned another type not like the other one and
called them the ones corresponding in course. And it
is that each pair of
8 zodiacal signs (has) one planet between them, such as
Aries and Scorpio to Mars, Taurus and Libra
9 to Venus.
10 And when Abū Maᶜshar transferred to the *Great
Introduction* the elements (of astrology) from
11 *al-Bizīdhaj*, he mentioned that the Persians called the
first type which is equipollent (*lit.* corresponding
12 in strength) potent, and the type which is correspond-
ing in ascension he called corresponding in
13 course, and he left the third type as it is.
14 And then Abū Muḥammad al-Saifī has mentioned it
and called the first type equipollent,
15 and he called it also corresponding in course. And he
judged Abū Maᶜshar (adversely) for calling
16 the second type the ones corresponding in course, and
he ascribed it to ignorance of the heavens(*or* circles,
or spheres, *manāṭiq*). And in spite of his (Abū
Maᶜshar's)
17 telling the truth, he (Abū Muḥammad) still degrades
Abū Maᶜshar, and he does not give him his due esteem.
For after all
18 Abū Maᶜshar does not deserve all this attribution of
ignorance, even though he has erred in nomenclature
19 here and followed partially the author of *al-Bizīdhaj*.

6:1 And had I been in Abū Maᶜshar's place, I would
have called the first type corresponding in
2 course as al-Saifī has done, because of the coinci-
dence of two transits of the azimuth(s) by the western
motion (as)
3 between each pair (of signs), and we treat them
according to one method. And then I would have called
the second corresponding
4 in times, or potent, because of the coincidence of the
two rising places. And I would have called the third
equipollent,
5 because power is more suitable for impressing and is
better for astrology.
6 But as for the eastern motion, the distance to the
ecliptic, the sun
7 and those of the fixed stars that have no latitude,
stay in it and do not leave it. And the fixed stars
8 which have latitudes (move) parallel to it (the
ecliptic) by it (the motion of precession). And the
six moving ones (i.e., the planets) pass through it
sometimes, and incline
9 from it the rest of the time towards the south and the
north. And because this eastern motion
10 is characterized by longitude, passing (or transiting)
through it (the longitude) is according to one of two
ways: First, passing by
11 the faster (one overtaking) the slower, either both in
(the same) direction or in two directions by (virtue
of) their difference
12 in retrogradation and forward motion.
13 And it was not the custom of the people to call
this a transit. But they express it as
14 conjunction or combust. And the second (way) is the
arrival of a planet at a (certain) time at a place
15 where another planet had been at a certain past time,
and so it is called transit
16 or passing its (the planet's) position. And this is
used in the transfers of the (cosmic) years. And it
is known

6:17 that there is no need for the longitudinal type of transit except advancement and delay only.

18 And here is another sense in which the astrologers use the name transit in which

19 (however) the sense of longitude is more legitimate. And that is if Saturn and Jupiter complete, in one

7:1 of the four triplicities, twelve conjunctions by their mean motions, they shift

2 to the triplicity which is next to that; they will conjoin in it twelve times also. And so they called

3 the shift, shift of the transit (<u>intiqāl</u> <u>al-mamarr</u>), and the beginning of the year in which (the shift)

takes place, (they called) the transfer (<u>taḥwīl</u>) of the transit.

4 The Property of Elevation

5 And associated with that transit is the property of elevation. It is used in longitude in

6 two ways, one is restricted and the other is absolute. The restricted one (is) a consequence of the westward motion

7 characterizing the horizon of an assumed abode (i.e. geographical position), and it is that the planet by it should be

8 in the tenth or the eleventh house, and it will rise because of its high position, there, over

9 all planets which are not in one of these two places at that time

10 and at that horizon, because elevation according to this restriction will be given to what is at the zenith,

11 and then (afterwards) to what is at the meridian, which is the extreme of the (body's) moving

12 by the westward motion, and the extreme elevation for a (certain) abode (i.e., locality). And the eleventh is preferred for its prosperity (<u>lit</u>. coming forward) to the

7:13 ninth for its adversity (<u>lit</u>. going away) and its declining, but if they become equally distant from

14 the meridian (sic, the sentence is incomplete.)

15 And as to the absolute one, (it is) also a consequence of the western motion not characterizing any

16 particular horizon. And this is why it is imagined to be a consequence of the eastward (motion). And it is that

17 the planet be in the tenth sign of another planet's sign, because the tenth (house)

18 is the most exalted center (<u>or</u> cardine) and the place of sovereignty and capture of everything else. So, on whichever horizon

19 this planet may be, by necessity it must continue in it, the first planet

8:1 will be in its midheaven elevated above it (i.e. the other planet).

2 And the specialists mix this absolute type with the restricted one.

3 And they express their two situations by motion of the strong one in the figure of the assumed horizon, and they use it

4 according to them because the seventh, though it is the tenth of the tenth, is then lower than it

5 and less than it in exaltation.

6 And thus, mention of (all) the possible types of transits that are related to longitude has been made.

7 Following is the latitudinal type of them, and elevation goes along with it, as well as with

8 thickness, from neither of which can it be separated.

9 Mention of Latitudinal Transit

10 And so we say regarding latitudinal transit, firstly, that the belief of the people concerning the northern region is

8:11 that it is the only known (region), although investiga-
tion has not shown it to be especially thus character-
ized positively,
12 except in a section where there are settled
habitations. As to the surface of the sphere of the
universe, it is, for all of it,
13 an elevation from all directions, and the sky is a
ceiling raised over the earth. And there is a point
in it (the sky)
14 assumed directly overhead for the inhabitants, to whom
it also has the highest
15 elevation, while in addition the remainder of the
sphere is (relatively) lower than it. But the northern
region is characterized
16 by human presence, and the zeniths of the inhabited
parts are in it. And the sun and the planets
17 ascend to it and descend from it, and hence they made
it elevation by position, and made the
18 planets with northern latitude above those with no
latitude or with southern (latitudes),
19 and the ones with more latitude to the north (above)
those with less latitude in it, and those with no

9:1 latitude above the ones with southern latitude, and
the one with less latitude
2 to the south above the one with more latitude in it.
And because the term elevation (<u>al-istiʿlāʾ</u>) has the
3 apparent connotation of sovereignty with no other of
the characteristics of elevation they used the name
<u>transit in</u>
4 <u>latitude</u>, and they said of the elevated one that it is
the one passing over the depressed one, meaning
5 by this difference a connection with the north pole
and by the one below, distance from it.
6 And the Hindus have an opinion regarding elevation,
though they did not mention transit in it. And that is

9:7 that their bases for it are in agreement with what we
related, except in (the case of) Venus. It is in the
south,
8 according to them, stronger than the north. And hence
its elevation is contrary to all other
9 planets, I mean, that in the south it is above one
with less latitude in the south
10 and one without latitude and one with northern
latitude; and in the north, (it is) above
11 one with more latitude. And so long as the (distance)
which is between the two planets
12 at conjunction is more than one cubit, and that is one
degree, they call it, in their language,
13 equality. And if it is not more than one cubit, they
call it fighting (<u>qitāl</u>) and warfare, and victory in
it,
14 with respect to position (is attributed to) the
elevated one, and with respect to power, (is to) the
one that has many testimonies and good fortune
15 according to their belief, but this is not the place
to mention it.
16 But what is necessitated by the eclipsing measure-
ment in which a planet passes over
17 another by the equality of their latitude in one
direction, (is) the nearest of the latitudinal transits.
18 Then that strength decreases according to the distance
between them, and then
19 the (above-)mentioned elevation occurs (while) one
transit is above the other.

10:1 But in opposition the strongest case in it (is)
the equality of the two latitudes with
2 difference of the two directions, but it is far from
the problem of the transit. And the nearest case in
3 an opposition in the case of the transit is the
equality of the two latitudes of the opposing (planets)
in one direction.

10:4 and that is by considering the eastward (i.e., proper) motion for both of them.

5 But if the westward (i.e., diurnal) motion is considered, it becomes necessity to substitute their declinations

6 for their latitudes. And if they become equal in one direction, their (daily) paths would be united,

7 and the transit of the planet would be at the position of the other; and if they differ by a (certain) magnitude,

8 the transit would be above its position or below it due to the difference of the two small circles (of declination).

9 And it is to this that the Hindus refer two times, which they believe are the extremeties

10 of bad luck. And their computation for it is mentioned in all of their <u>zījes</u>.

11 And they are: (first) the times when the two luminaries are on one small circle (of declination) when the sum of the distances

12 of their true longitudes from the beginning of Aries (is) six whole signs, and when they are on two equal small circles

13 when the sum of the distances of their true longitudes from Aries is equal to twelve

14 whole signs. And this (is so) if the moon has zero latitude. But if it has

15 a (non-zero) latitude (it will be) when it is on the small circle (of declination) of the sun or the (one) equal to it by observation, (i.e., parallax included)

16 not by computation.

17 And Muḥammad ibn ʿAbdullāh ibn ʿUmar al-Bāzyār has said, in the beginning of the sixth treatise

18 of the <u>Book of Conjunctions</u> that every heavenly body

19 is higher than the one following it in rank. And it is shown by the passing of one

11:1 over the other that if one of them is covered by the other, then their longitudes,

2 latitudes, ascents (<u>ṣuʿūd</u>), and descents (<u>hubūṭ</u>) in one direction are equal at the equinox, that

3 would be a cause for the eclipsing of the lower (one by) the upper (one), and that would be an indication

4 of lower (i.e. earthly) incidents. And it is apparent that his words stand for latitudinal transits.

5 (and this) cannot be interpreted otherwise. And in it mention of equality in longitude and latitude replaces

6 mention of ascent and descent. And the equality of the magnitude of the two latitudes when the

7 two longitudes are equal, necessitates an eclipse of the planets exactly,

8 (though) perceived by his eye with a parallatic difference.

9 And then he took up the transit in thickness as an example, but with no success(?).

10 And after this example he said that the strongest indications of the high bodies (i.e. planets) at the passing of some of them

11 over others appear at conjunction, whereas in oppositions and quadratures

12 and the other aspects their indications will be less apparent.

13 And if he meant by it the transit in thickness, he also points his eye towards the

14 latitudinal transit, and this is the one to be considered. And verily he said, is it not that if two planets ascend in one direction

15 and their parts (longitudes) are equal, the one that rises first has the power, and that will not be

16 except by its reaching the (afore-)mentioned elevation first, I mean that its latitude will increase toward the north

17 and decrease toward the south,

11:18 Reference to the Order of the Planetary Spheres

19 And there remains of the division its third part,
and it is the objective of what we (now)

12:1 discuss, I mean the transit in thickness. And to this
the astrologers refer
2 in the conjunctions of Saturn and Jupiter, and they
call it a transit as a convention among themselves.
3 And had it not been for this it was known that the
people of this craft are agreed among themselves that
4 the nearest sphere to us is the sphere of the moon and
the farthest of the spheres of the planets
5 from us is the sphere of Saturn. And if they said,
regarding the transit of the moon, that it is above
Saturn, it was denying
6 their saying that one planet, the extreme distance
from the earth of which is sixty-four
7 times its (the earth's) radius, passes over another,
the nearest distance of which from the earth is
8 fourteen thousand eight hundred and eighty-one times
its radius. But it is an expression
9 without leading to this meaning, which is well-known
among them by agreeing on it by convention,
10 although the order of the planets is not necessarily
thus.
11 And I do not mean by that the confusion arising
from one who is not in
12 the profession and (is not) of its people, such as the
sectarian talk among the Hindus regarding the moon (to
the effect that) it is
13 above the sun, and like the laity among the other
(people), who ascribe motion to the planets
14 in comparison to the stillness of the sky above them.
And such opinions are those of the uneducated,
15 and have no relevance (as is the case) in any craft
between one who clothes himself (in the profession)
and one who divests himself of it. (Just) as

12:16 there is no (useful) outcome from a talk between two
persons one of whom speaks a language not understood
by the other, thus also
17 the opinions of those on the inside and those outside
are at the extremes of contradiction. But what I mean
(are)
18 disputes which occur among the people of the profes-
sion, who are industriously engaged in investigating
it,
19 which (disputes) do not prevent them inquiring and
expounding because of pride. And these had realized

13:1 the elevation of the sun above the moon, and they lower
the moon from it (the sun), and (they determined) the
magnitudes of their distances from
2 the earth, and ascertained their farthest distance,
mean distance, and nearest distance from the earth.
3 And they ascertained the ratios of the nearest
distances of the planets to their farthest distances
4 only, without the absolute distances.
5 And some of the Persians placed the moon and
Saturn at the two ends of the ether
6 because the days of the cycle of one of them are near
to the years of the cycle of the other. And then they
placed the sun
7 and Jupiter as the next (planets) from the two ends,
because of the equality
8 of the months of the cycle of this to the years of
that, approximately.
9 But this correlation which is taken from the times
did not turn out to be so after that.
10 And so they placed the sun at the center of the
epicycle of Venus, and they placed Mercury and Mars
11 above it so that the height of Mercury above the sun
became as the lowering of Mars beneath Jupiter.

13:12 And they ascribed this opinion to that part of the Avesta, the religious book of the Magians,

13 which was transferred to the Byzantines (or Greeks) by Alexander.

14 And some of them (i.e., the Persians?) made the sun a center for the epicycles

15 of Venus and Mercury, and they made the three superior ones above that, according to their order.

16 But the Greeks were so suspicious that

17 Plato doubted whether Venus is below the sun or

18 above, as was told by Yaḥyā the Grammarian (John Philoponus) in his refutation of Proclus.

19 Thereupon the sagacious of them have accepted, regarding their (the planet's) motions,

14:1 the putting of all the planets proper (i.e., without moon and sun) above the sun;

2 they were left with (a space) between the spheres of the two luminaries, devoid of a planet. The space was occupied by

3 the two separated planets (Mercury and Venus) rotating around the sun at a fixed distance.

4 Its (the space's) thickness is not less than the thickness of the two spheres according to their minimum

5 and maximum distances. And in it (the space) nothing impossible or prohibited occurs, such as intermingling, collision, or hindrance.

6 And so they considered the sun as being in the middle with three of them which are lower than it and three above it,

7 according to the solar arrangement.

8 And the learned among them found this a good opinion and preferred it (to the others?) and none

9 of the astronomers of the nations have contradicted them. For indeed most of them use the names of the week days by the names

10 of the seven planets as fixed by what is required by the lords of the hours

14:11 which are taken in this order going from the highest of the planets to

12 the lowest.

13 And since this is the most widespread opinion and one relied upon by all,

14 so the people's expression in (saying) transit above or below, this is from a special (i.e., technical) meaning for them, so let us go back to it.

15 Mention of the Three Distances

16 In the Eccentric Orbits

17 But let us introduce before it (the transit), the distances of the planet and its variation in its sphere, and what follows

18 from that with regard to ascent and descent and their consequences, so that it will be easy to get

19 what comes after it.

15:1 And we say that each one of the planets is characterized with

2 respect to the eccentric heaven, whether it be the deferent or the

3 epicycle, by distances from the earth which vary between a greatest and a smallest, its two extremes (of distance),

4 and a mean which is necessarily fixed between them. Hence, the fixed distances of the planet

5 from the earth are three: the nearest, the mean, and the farthest. And the mean is not (at)

6 one (position) for either of them (i.e., the deferent or the epicycle), but it is (at) two (positions) on the sides of the diameter passing through the farthest and the nearest (distances)

7 one at its right and the other at its left. But mention of one of them is left out

15:8 in enumerating them due to their equality and the agreement of the situations in them. And therefore each one of the two heavens,
9 the deferent and the epicycle is divided by the (above-)mentioned distances into four pieces which in fact are
10 the sectors (al-niṭāqāt). The two (sectors) of them which are elevated are called ascending, and the two
11 below, (are called) descending, and that is (reckoned) by addition to the mean distance to the planet.
12 In one of the two ascending (sectors) it (the planet) is coming down, and in one of the two descending ones, (it is) ascending.
13 And the Persian astrologers call what relates to the deferent (lit. the heaven of the apogee), jawwī
14 and they say that the planet is ascending in the jawwī or descending in it. And they call
15 what relates in this matter to the epicycle a chord, and so they say that it is ascending in the chord
16 or descending in it.
17 But as for the chord, in its less used sense, it is the sense by which the zīj (i.e., a set of astronomical tables)
18 is called a zīj due to the discussion of chords in it. But the heaven of the apogee is not distinguished
19 from them (chords) in it (the zīj).

16:1 As for its primary sense, it is the opinion of the ancients regarding the halters (or bonds) of the planets
2 with the sun and their retrogradation from the tension of the cord tightened by it, and its forward motion
3 by its slackening. And since that was, according to those of them who investigated retrogradation and forward motion,
4 the epicycle, they referred its cases to the cause (which is) well-known among the masses of them, namely among the majority of them, which is

16:5 the binding cord (=chord). And there is no difficulty with nomenclature and titles, as long as they do not corrupt the meaning.
6 And the way to the more fitting and proper (of the names) for it (the meaning) is straightforward, unhindered.
7 However, as for the name jawwī for the deferent, I cannot find an explanation for it, since they
8 call right ascensions jūyi rāst, and one understands by this name (i.e., jawwī) either one
9 of the two meanings, but its place is in it (? i.e. the meaning is fixed by the context?).
10 But (the word) kura (sphere) has been arabized (from another language) and in Persian it is kūi, And one understands by attributing to it (i.e. jūyi rāst)
11 also rightness (or straightness), one of two things: either straightness of the tables, which is the farthest of the two (meanings from actuality), because
12 the tables of oblique ascensions, yea all tables, (are) straight in planning. By
13 this, the thought that jawwī is the table is weakened. But if one has to explain the meaning of the "straight" table, then its
14 straightness is the consecutiveness of what is in it due to the weakness of the day of each locality in
15 the same way.
16 As to what is meant by calling the terrestrial equator the right sphere,
17 (it is) due to the lack of small circles in it. And it is (also) possible to call it a river due to the sailing of the planets
18 in it like the sailing of ships. And the most probable of (the meanings) we enumerated is that jūyi rāst is the right sphere,
19 in which case the deferent has nothing to do with that (i.e., the word jawwī), since it (the deferent) does not resemble

17:1 the epicycle as to sphericity except by its surrounding the earth. So it is supposed that jawwī

2 and watar (chord) are two restricted (i.e. technical) titles by which two things are referred to where what is meant by them is known.

3 And let us mention the names of the three distances in each one of the two circles,

4 according to the people of the profession. As for the farthest distance in the deferent it is called, in the Greek language,

5 afrabkhiyūn (sic), and in the Hindu, uuj and it is by this that it is known and used. And its meaning in their language

6 (is) loftiness and height so that they called the most exalted planet like that

7 aujast. And their scientists call the farthest distance mandūj with the addition of the meaning of looking

8 at it, because they call the epicycle sīkrā'ī, the fast (one).

9 And what made them do that is that attainment of (a cycle of) the equation due to

10 the deferent will be in a slower time than attainment of the one due to the epicycle.

11 And that is common to the five planets (proper), because they do not see in the motions of

12 the moon anything that requires an epicycle for it. And that may be because

13 revolution in the epicycle, in cases other than that of Venus, (is) faster than revolution

14 in the deferent.

15 And Ḥamza ibn al-Ḥasan al-Isfahānī claimed in the "Book of Contrasts" (Kitāb al-muwāzina) that apogee

16 is the arabized form of auk which means, in Pahlavi Persian, elevation and loftiness. As to the nearest distance

17:17 in the deferent it is called in the Greek language afranjiyūn, but I did not hear from the Hindus a special name for it.

18 And linguistic analogy gives it as bahalā(?), because it is sinking low and degradation.

19 And as to its name (here a gap in the ms.) the exaltation of the planet nīḥ(?) because they call its descent

18:1 nījast(?), but Ḥamza did not mention it. And the folk of the Arabic language (meaning those having Arabic as their native tongue) call it, when they need to

2 mention it, sometimes the counter(point) to the apogee and sometimes the opposite (point to) the apogee, and sometimes,

3 ḥaḍīḍ (the common word for perigee, lit., the lowest category).

4 But as to the mean distance, it does not have a special name, so far as we know. And let us (now) go back

5 to the epicycle. The farthest distance in it is called in Arabic al-dhirwa (the epicyclic apogee), and in the Greek language

6 (it is) as what was aforementioned for the deferent, referred to the fiqilus, it being the epicycle. And the

7 nearest distance in Arabic is the counter(point) of the dhirwa or the perigee of the epicycle, and in Greek, analogously

8 to the previous, afranjiyūn fiqilus. The mean distance is (used) as it is with

9 (the word) epicycle adjoined. But as to the apogee and the perigee in the eccentric circle,

10 they do not differ visibly, because the line passing through its (the deferent's) center and that of the universe

18:11 is one straight (line). However the mean distance differs with it.

12 And let us draw for that a picture that will make it (easy to) sense. So let ABJ (in Figure 1)

13 be the parecliptic with center H which (is) in fact the center

14 of the universe. And our position is on it (H) approximately, because there is no sensible difference between the two of them and no apparent magnitude.

15 And let HD (be) the quantity between it and the

16 deferent center.

17 Ptolemy showed in the third treatise of the Almagest that

18 the difference present for the course of the planet due to this heaven will be the same

19 whether it is smaller than the parecliptic or bigger than it or equal to it. And the smaller is like

19:1 AḤṢ, and whether it is tangent to the parecliptic at A or it differs from it, the place of their conjunction

2 (is) the apogee, I mean point A, and the perigee, point Ṣ. And their positions in

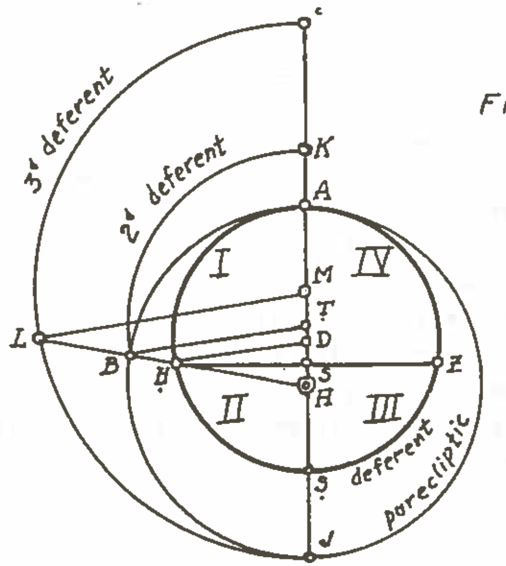


Figure 1
(p. 21 of text)

19:3 the ecliptic are what join them (the positions) to the (line of) centers, which coincides with the diameter ADHJ.

4 And then we bisect DH at S and drop upon it chord ḤSZ perpendicular

5 to diameter AHJ, and the two points Z (and) Ḥ will be for the two mean distances.

6 And because if we join DḤ (and) ḤH the two sides DS

7 (and) SḤ will be equal to the two sides HS and SḤ, and the two angles DSḤ (and) HSḤ

8 (are) right angles, so the two bases DḤ (and) ḤH are equal. And hence ḤH is

9 equal to the radius of circle AḤṢ and the farthest distance, which is HA,

10 exceeds the radius of that circle by the eccentricity, I mean HD. And the nearest distance,

11 which is HṢ, is less than half it (half of AṢ) by the eccentricity, and the mean distance

12 is the one equal to it (i.e. DA), and that is half the sum of the two adjacent distances.

13 But the (angular) distance of point Ḥ, which yields the mean distance, from the apogee is found at

14 the center of the eccentric orbit to be angle ADḤ, but at the center of the universe

15 angle AḤH; and angle ADḤ, which is for the middle (distance) of the travel, exceeds angle AḤH,

16 which is for the true position and the way it is seen, by angle DḤH, which is for the equation. And likewise is

17 the situation at point Z, which is for the other, right(-hand) mean distance.

18 Thus it is determined that the attaining of the mean distance, (starting) from the apogee, by the mean motion

19 (is) more than a quadrant, and by the unequal motion (is) less than a quadrant.

20:1 And we extend HḤ to B on the parecliptic.
2 And B will be the place of intersection of the two circles (namely), the apogee (circle) and the parecliptic, which are equal
3 at the middle one. And it is the one preferred by Ptolemy, for elegance(?), not that he was obliged to.
4 So let KB be a segment of this eccentric orbit, and let us produce
5 BṬ parallel to ḤD, and Ṭ will be the center of the apogee circle (or a new deferent),
6 because the ratio of DH to DḤ is as the ratio of HṬ to ṬB, due to the similarity
7 of the isosceles triangles DḤH and ṬBH. And B is at the mean distance.
8 And let ᶜLJ be (half another) assumed deferent, greater than
9 the parecliptic, either tangent to the parecliptic at J or differing from it (completely). And we produce HḤB
10 to L and extend LM parallel to BṬ,
11 and M will be the center of ᶜLJ similarly to what preceded. And because DH is equal
12 to the sine of the maximum equation, hence the determination (of the distance) between (the center of) the apogee (heaven) and (the projection on the apsidal line of the position at) mean distance
13 will be to halve the sine of the maximum equation, and that is DS, and its
14 arc (sine) is then taken and added to one quadrant of the circle, which is the arc (sine of) AD, and the sum will be arc AḤ,
15 which is what was required. And if it is subtracted from the circumference, there remains arc AHSZ,
16 (which is) the (angular) distance of the other mean distance (position) from the apogee, and from it (the apogee) is measured the unmodified argument (or anomaly)

20:17 for the sun, and (also) for the moon according to the belief of the Hindus, and Hipparchus, and the (other) ancient
18 Greeks. And in the (case of) planets the unmodified (i.e. mean) longitude, called the center, is measured from it.

21:1 And Kūshyār ibn Labbān followed the same procedure in his Jāmaᶜ Zīj, in
2 the case of the sectors, and he considered them according to mean distance. But he added, for the first sector,
3 to the quadrant, one half the greatest equation. And this increase is greater than that (proper) magnitude, as
4 is evident from the difference of the sines and the mention of their degrees. But the halving (should properly) occur with the sine
5 of the maximum equation. It is as though he had followed Abū Maᶜshar, who did the same in
6 the thirty-eighth chapter of his zīj, and he confirmed it afterwards.

7 Mention of the Distances of the Mean Planets
8 in the Heavens of their Apogees

9 And it is apparent that the crux (? madār) of that depends on the quantity of the maximum equation, and (these) differ

22:1 in the zījes for (various) reasons, the most important among which are the difference(s) due to instruments
2 and operations, yet this is not the proper place for talking about that. Other differences are due to some other reasons, some of which will become apparent
3 when we (now) talk about the two luminaries.
4 So we say that Ptolemy mentioned in the Almagest that he found it

22:5 (to be) two parts and twenty three minutes. He was followed in this by Theon of Alexandria
6 in the Canon; and inasmuch as his (Ptolemy's) procedure, in the observations from which he ascertains the eccentricity,
7 is not reliable, the (above-)mentioned computation is unsure.
8 But as to the maximum equation of the moon, he found it to be five parts. After him
9 the maximum equation of the sun was found, in the reign of al-Maʾmūn, by Yaḥyā ibn Abī Manṣūr, to be
10 one part and forty seven minutes, and this observation is not reliable, according to what is said of it in
11 the reports. Khālid ibn ʿAbd al-Malik al-Marwarūzī found it, under the supervision of Sanad ibn
12 ʿAlī, to be less than two parts by six seconds. Ḥabash has put it in his zīj according
13 to the observation of the Banī Mūsā ibn Shākir, as less than two parts by one minute. It was found
14 by Muḥammad ibn Jābir al-Battānī (to be) less than two parts by fifty seconds, and we found it
15 to be near to this quantity. And through his own observation Abū al-Wafāʾ al-Būzjānī found it to be one part
16 and fifty-nine minutes, once, diminished by two seconds and another time fifteen seconds, and once
17 increased by seven seconds, and another time two seconds and twenty thirds.
18 And that, due to variations in observation and computation, Abū Ḥāmid al-Ṣaghānī has found (to be)
19 more than two parts by one third of a minute by using sines, and when he calculated it using chords

23:1 and observation, he found it as exceeding two parts by six minutes and six seconds. And it was put by
2 Abū al-Qāsim ibn al-Aʿlam al-ʿAlawī in his zīj called al-ʿAḍudī as greater than two parts

23:3 by one sixth of a minute, and he was, as he told me, planning to make instruments,
4 and conducting observations. It was found by Abū Dāʾūd Sulaymān ibn ʿIṣmat al-Samarqandī to be
5 one part and fifty-five minutes and two seconds, but he had used in deriving it
6 Yaḥyā and Ptolemy's method of observing the times of the solstices, and that is theoretically correct
7 but invalid in practice. Abū Muḥammad al-Nasafī(?) put it in his al-Mukhtaṣar (summarized) Zīj as greater than what
8 Ptolemy had by four minutes. He pretended(?) that he had made observations while, in fact, he is a plagiarising liar and an impostor to
9 the craft (of astronomy).
10 The moderns have not, to all appearances, made observations on the moon by way of checking,
11 since none have appeared either differing or in agreement, and they all follow in its single equation (i.e. the one independent of the sun)
12 either Ptolemy in that it is five parts and one minute or Theon in dropping out the minute.
13 And I have not seen on this subject anything other than what is in Ibn al-Aʿlam's zīj where his equation
14 is less than five parts by seven minutes. But strangest of all is the case of Muḥammad ibn
15 Isḥāq al-Sarakhsī, who follows Ptolemy with regard to the magnitude of this equation although he
16 is one of those who follow the Sindhind.
17 I have read in the commentaries of al-Jaihānī that the equation
18 of the sun in the Maʾmūnic (zīj), which is one part and forty-seven minutes, if one half of its seventh is added to it
19 it will be equivalent to what was found by Sulaymān, and if one seventh of it is added to it, it will become what was found

24:1 in Damascus, and if its two ninths are added it will be equal to what is in the Sindhind Zīj, while if
2 its fourth is added to it, it will become equal to what is in al-Khwārizmī's zīj, and if its third is added to it
3 it will become equal to what is in the Almagest, and this will be so after rounding off the seconds to minutes by taking
4 as one minute the number of seconds that is greater than half a minute. The (above-)mentioned people practised observations,
5 and the differences among them (are with) regard to the (actual) positions. And as for those who do not refer (to anything) but
6 what is good, as (do) the Hindus in describing the situation with regard to themselves, (let them make) some verification of the section (here) discussed.
7 We say that they originated the maximum equation of the sun which is
8 two parts and fourteen minutes, and the maximum equation of the moon which is four
9 parts and fifty [-six] minutes, and they (the equations) are thus in the Shāh Zīj, since it has passed from
10 India to the Persians, and this is why they were put thus in Abū Maᶜshar's zīj since
11 he depended on the Persians. But most of their zījes are based on approximations, and in them they
12 obtain some magnitudes from other magnitudes, and they resort for that to
13 the total sine. It resembles getting the latitude of the moon from the sine by multiplying the sine of its distance
14 from the node by nine and dividing the product by five, since this is the ratio of
15 the maximum latitude of the moon to the sine of the maximum distance, it being the total sine if

24:16 it (R) were two and one half parts, and the maximum latitude of the moon four and one half parts.
17 As to the amount ascribed to the Sindhind, with the addition of Yasᶜā al-Ma'mūnī to it,
18 it is two parts and eleven minutes. And it is this that al-Fazārī made use of
19 in subtracting from the sine of the argument of the sun [an eighth] of it, and in doubling the sine of the argument of the moon

25:1 to obtain their equations. And thus the maximum equation of the sun comes out equal to two parts
2 and eleven minutes and one fourth of a minute, and that for the moon equal to five parts, and that (is) as though
3 the total sine were one hundred and fifty minutes. But had he used in
4 the case of the sun the method of subtracting the ninth instead of the eighth he would have been nearer to the opinion of the people, and
5 others would have done that.
6 It is found in some of the copies of the Shāh Zīj that the number of the minutes in the equation of the sun
7 is thirteen, and thus (also), in the equation of the moon, if from twice the sine
8 a seventy-fifth of it is subtracted that would have been nearer to that quantity.
9 And there was mentioned in some of the books a story about al-Fazārī regarding the equation
10 of the sun, where he multiplies the sine of its argument in the <u>kardajāt</u> of the Sindhind by a hundred
11 and five and divides the quantity by 2616, and that (sine of the argument) in the <u>kardajāt</u>
12 of Āryabhaṭa by seven and divides the result by 180. And in the case of the equation of the moon he
13 multiplies the sine of its argument in the <u>kardajāt</u> of the Sindhind by ten and divides the product

25:14 by a hundred and seven, and that (sine of the argument) in the kardajāt of Āryabhaṭa by ten, and divides the product by
15 one hundred and seventeen, and the treatment of the total sine along these lines is as preceded, after
16 it is known that these kardajāt for a quadrant of a circle are twenty-four, and each
17 one of them is three parts and one half and one quarter.
18 And the total sine for Āryabhaṭa is three thousand four hundred and thirty-eight
19 minutes, and with this the maximum equation of the sun will come out as two parts, thirteen

26:1 minutes, and forty-two seconds, and by rounding off the seconds we end with what is required.
2 And the maximum equation of the moon will come out as four parts, fifty-six minutes,
3 and twenty-three seconds. If (the seconds) are deleted the remainder will be what is required.
4 And it is to this that the author of the Harqan Zīj refers, which is written in poetry after
5 the Hindu way of writing science in sloka verses. So, when he used the sine of
6 Āryabhaṭa, he said, regarding the equation(s) of the two luminaries:
7 "And if you come upon something, add it to the sine
8 "Not in it, and then from the sine you wanted,
9 "Then multiply it by 7 ([z]a'), and take pleasure in working skilfully,
10 "Then faq it (i.e., divide by 180) to obtain the result.
11 "It is accurate rīsāt if you computed (correctly).
12 "And then you drop every sixty, as you used to do,
13 "And thus does the learned man not(?) on each occasion,
14 "Except that the 180 (faq) is for the sun, and with 116 (wīq) for the moon.

26:15 "And for each, God has given a measured share."
16 This is a section (which) sets forth in its first (part) the determination of the sine of the argument, and it prescribes multiplying it by
17 seven, it is the [z]a', and dividing the number resulting by one hundred and eighty, which is the faq, and there come out
18 minutes, which are the rīsāt. Elevate them into degrees, and they are the ratios not yet rounded off (text garbled).
19 The part of division in the (case of the) moon is one hundred and sixteen, it being [wīq].

27:1 As to the kardajāt of the Sindhind, which (latter) is the Brahmasiddhānta, its
2 author, Brahmagupta, has put the total sine in it as three thousand two hundred
3 and seventy minutes, from which the equation of the sun will come out, according to the (above-)mentioned operation, as two parts,
4 ten minutes, and twenty-nine seconds, and the equation of the moon, five parts. So, verily
5 the cause of the difference(s) in the maximum equation among the Hindus has become evident; it is due to
6 the total sine and the variation in what was taken for it, ignoring observation.
7 But that becomes clearer by enumerating what is in their zījes about it.
8 We say that [Nābhāla] the Brahman put in his zīj the kardajāt of Āryabhaṭa and prescribed for
9 the equation of the sun what was related previously in (our) account.
10 As to the equation of the moon, he prescribed multiplying the sine of its argument by thirty-one
11 and dividing the result by three hundred and sixty. And by that his maximum equation will come out
12 as four parts, fifty-six minutes, and three seconds.

27:13 As to the Karaṇasāra, meaning "Breaker of the Zījes", its author
14 Vittesvara, prescribed in the case of the sun multiplying by ten and dividing by twenty-three,
15 and in the case of the moon, taking the sine as it is, without multiplication or division.
16 And the total sine in these kardajāt is three hundred minutes. So it is apparent that the maximum equation of
17 the sun will come out by this as two parts, ten minutes, and twenty-six seconds.
18 And the maximum equation of the moon is five parts in the Karaṇatilaka meaning, "The Choice Part of
19 the Zījes" (lit. "The Forelock of the Zījes"). Its author, Vijayanandin, prescribed, in the case of the sun, [multiplying by two and dividing by three, and in the case of the moon] multiplying by three and dividing

28:1 by two. The total sine in his kardajāt is two hundred minutes, and this is why the maximum equation of the sun has come out
2 equal to two parts, thirteen minutes, and twenty seconds,
3 and that for the moon as five parts.
4 And there are found in some works which are more precious than their zījes, namely the siddhāntas, numbers for the two luminaries
5 which are called circumferences (muḥīṭāt) which are to be multiplied by, and other numbers with them which are
6 the parts of the division. Thus, in the Pulisasiddhānta the circumference (muḥīṭ) of the apogee of the sun is fourteen
7 parts, and the circumference of the apogee of the moon is thirty-one parts.
8 And in the Brahmasiddhānta, a tale without display of the (actual) operation, the circumference

28:9 of the apogee of the sun is equal to thirteen parts and forty minutes, and the circumference of the apogee of the moon is thirty-one
10 parts and twenty-six minutes. And the meaning of this circumference is that if they sighted, on the center of the circle of the apogee
11 and with the distance of the sine of the maximum equation, which is
12 the eccentricity, a circle, and they called it the circumference of the apogee, and for that they have (certain) reasons, in their operations,
13 the explanation of which would involve a lengthy discussion.
14 And since the ratio of the circumference to the diameter, according to Paulus, is as the ratio of three thousand
15 nine hundred and twenty-seven to one thousand and two hundred and fifty, the radius of
16 the circumference of the apogee according to this ratio will be, according to Paulus, for the sun, two parts and thirteen minutes
17 and forty-one seconds. And for the moon, four parts, fifty-six
18 minutes, and one second, and on rounding off and truncation we get what we have mentioned.
19 But he points out that the ratio of these circumferences to the maximum equation

29:1 is as the ratio of the circumference, which is three hundred and sixty, to the total sine, and if
2 we derive the circumferences from the equations of the two luminaries by this ratio, they will come out as
3 fourteen parts and three minutes for the sun, and thirty parts, fifty-nine
4 minutes, and forty-one seconds for the moon, and those are their apogees (i.e. apogee epicycles).
5 But according to Brahmagupta the square of the diameter is one tenth of the square of the circumference, and accordingly

29:6 the equation of the sun will be by this ratio two parts, nine
7 minutes, and nine and two thirds of a second, and the equation of the moon will be four parts, fifty-eight
8 minutes, and twelve seconds. And if we compute the circumference of the apogee assuming that its ratio
9 to the maximum equation is as the ratio of the circumference to the total sine, using the quantity at which he
10 estimated it, they will come out for the sun as four-teen parts and forty-five minutes, and for the moon
11 [thirty-two] parts, and thirty-five minutes and twenty-seven seconds,
12 and both of them are sharply in disagreement with what we said. So this is the situation with the equations of the two luminaries.

13 And as to the equations of the five planets in the deferent, Theon
14 has followed in most cases the Almagest, but he has for Saturn in his zīj, the Canon (al-Qānūn), six parts
15 and thirty-one minutes, which is one minute less than that of the Almagest, and for Jupiter five
16 parts and fifteen minutes, which is also one minute less, and for Mars eleven parts,
17 and twenty-five minutes, and for Venus two parts and twenty-three minutes,
18 and for Mercury three parts and two minutes, which is ten minutes more (than that of the Almagest).

19 And the majority of the moderns have followed him because they did not make any

30:1 observations on them, and so they did not change them, except for Venus. And the agreement among them is
2 that the equation of the sun is equivalent to its (Venus') equation. And they observed the sun and thus took for
3 its (Venus') equation the same as its (the sun's) equation.

30:4 But in his zīj, Abū al-Qāsim ibn al-Aᶜlam diminishes (from the Almagest?) forty-eight minutes in the case of Saturn,
5 and increases eighteen minutes in the case of Jupiter, and diminishes
6 twenty-three minutes for Venus, and increases thirty-eight minutes for Mercury.
7 But the only justification for such (things) is their presence (in the text), but the criterion for accepting them is a display of the operation,
8 as was done by Ptolemy. But this is not found in the case of any of the moderns, and thus
9 the accusation against their operation is reinforced.

10 As to the Hindus and the Persians, they have a common opinion, and so the
11 zījes of the Shāh, and Abū Maᶜshar, and Yaᶜqūb ibn Ṭāriq contain nothing on which they differ except only one thing,
12 the difference of which does not exceed one minute. But Muhammad ibn Mūsā al-Khwārizmī
13 lacks this (agreement) in his zīj. And they have for Saturn eight parts and thirty-seven
14 minutes, and for Jupiter five parts and six minutes, and for Mars eleven parts
15 and twelve minutes, and for Venus two parts and thirteen minutes, and for Mercury
16 four parts.

17 But al-Khwārizmī adds to Mercury two minutes, following Theon in this, but
18 differing (from Theon) in the integer part, following for it the Hindus, as if he is to decide which part to choose from which!
19 And the law of al-Fazārī is proportional to these quantities. He suggests in the case of Saturn

31:1 multiplying the sum of the sine (of the argument) and its tenth and one sixth of its tenth by three.

31:2 and for Jupiter, to double the sum of the sine and one
fifth its tenth, and for Mars,
3 to multiply the sum of the sine and its tenth and a
sixth of its tenth by four, and
4 for Venus to diminish from the sine one tenth of it,
and for Mercury to add to the sine
5 three fifths of it. And if we consider this with the
total sine which is according to him, a hundred
6 and fifty minutes, for deriving the maximum equations,
there results: for Saturn 8;37,30,
7 and for Jupiter 5;[6], and for Mars [1]1;10, and for
Venus 2;15, and for Mercury
8 4;[0].
9 However, Muhammad ibn Isḥāq al-Sarakhsī combined
both opinions. For he took the equation
10 of Saturn from the Hindus, and that of Jupiter and Mars
from the Canon, and that of Mercury from al-Khwārizmī.
11 And he added one minute in the case of Venus to what
is in the Canon, and the reasons for doing this are
not apparent,
12 for showing it (the method of derivation) is necessary
for acceptance (of the results), as he(?) did in
increasing
13 the cycles of Saturn in cosmic days, and thus was near
to the opinion of the Hindus in that respect, and what
our associates have
14 for it (the cycles?) is from it (the increase?),
although this (which our associates have) was
erroneously reported in the Sindhind zījes.
15 And what is in the Hindu zījes which we have read
is quite confused,
16 to the extent that it is unacceptable (as being the
authors' fault), so that the accusation falls upon the
copies
17 at hand and (upon) the translator who dictates to us.
That is that Paulus has announced the magnitudes of
these

31:18 equations as: for Saturn, in minutes, 568; for Jupiter,
284; for Mars, 676;
19 for Venus 134, for Mercury 268, and (he) divided its
product by three hundred and sixty by

32:1 3438 minutes, the total sine, so that the circumferences
of the apogees came out for Saturn [60], Jupiter
2 30, for Mars 70, for Venus 14, and for Mercury 28.
3 And those among them who compared the sine and
the equations, like
4 the author of the <u>Karaṇatilakā</u>, the total sine
according to him being two hundred minutes, he
suggested in the case of Saturn
5 multiplying half the sine by three and adding to the
result its sixth, and if we do
6 that for the total sine, the maximum equation for it
will come out, 5;[10]; and for Jupiter
7 to multiply the sine by three and halving the result
and adding to it one sixth of a tenth (of it),
8 and so we get for it 5;5, and for Mars to multiply the
sine by three and to add
9 to the result (its) seventh, and it will come out as
11;25, and for Venus to add to the
10 sine its sixth and to take half the result, and there
will be for it 1;[5]6,40, and for
11 Mercury to multiply the sine by three and to halve (it)
and to diminish it by a tenth,
12 and there will be for it 4;30. And what comes out
from the circumferences which Paulus put
13 is: for Saturn 9;33,
14 for Jupiter 4;46,30, for Mars 11;[8],30, for Venus
2;[13],42, and for Mercury 4;27,24.
15 As for those who set up ratios between the solar
equation and these equations, such
16 as the translator (<u>or</u> commentator?) of the <u>Khaṇḍakhaḍ-
yaka</u>, which is known to us as the <u>Arkand</u>, he claimed

32:17 that the equation of Saturn is four times the sum of the equation of the sun and [half] its seventh,
18 and the equation of Jupiter twice the equation of the sun and one time its seventh, and the equation
19 of Mars five times the equation of the sun, and the equation of Venus as its equation,

33:1 and the equation of Mercury twice its equation.
2 And what comes out by these maximum equations is near to what came out
3 from the circumferences which were put by Paulus.
4 Verily I have seen Awlath ibn Sahāwī, the astrologer, using the equation of the sun
5 in those circumferences instead of the sine, and multiplying it by them (the circumferences), and he divides the result in all (cases)
6 by fourteen, and it comes out near to what comes out from the sine, whereas if he put the circumference
7 of the apogee of Saturn as fifty-four instead of sixty, and the circumference of the apogee of Jupiter as thirty-two
8 instead of thirty, and the circumference of the apogee of Mercury twenty-five instead of twenty-eight,
9 and then used in it the equation of the sun, there would have resulted what
10 is nearer to what is generally agreed upon, according to what we said. It is because some of the eccentric orbits
11 are not fixed in position, due to the motion of their centers along the circumference
12 of the circle carrying them, like (the case of) the moon and Mercury according to Ptolemy, and their mean distances
13 are not as well fixed in position, nor are their recedings from the apogee fixed
14 at one value.
15 To understand that let us assume the center of the universe to be H (Figure 2) and the circle

33:16 on whose circumference moves the center of the circle carrying the epicycle of the moon be circle
17 ẒT with center at H. We describe about it (H), at a distance half the diameter of the deferent, circle
18 BḤJ. Let the center of the deferent be D, and half its diameter DA.
19 So HA will be the farthest distance, the sum of DH (and) AD, and the nearest distance (will be)

34:1 the difference between them. So the mean distance is necessarily DA. And let the intersection of the
2 deferent with circle BḤJ, which is Ḥ, be the mean distance at the time.
4 And it is evident from what has passed that the perpendicular ḤS to BJ

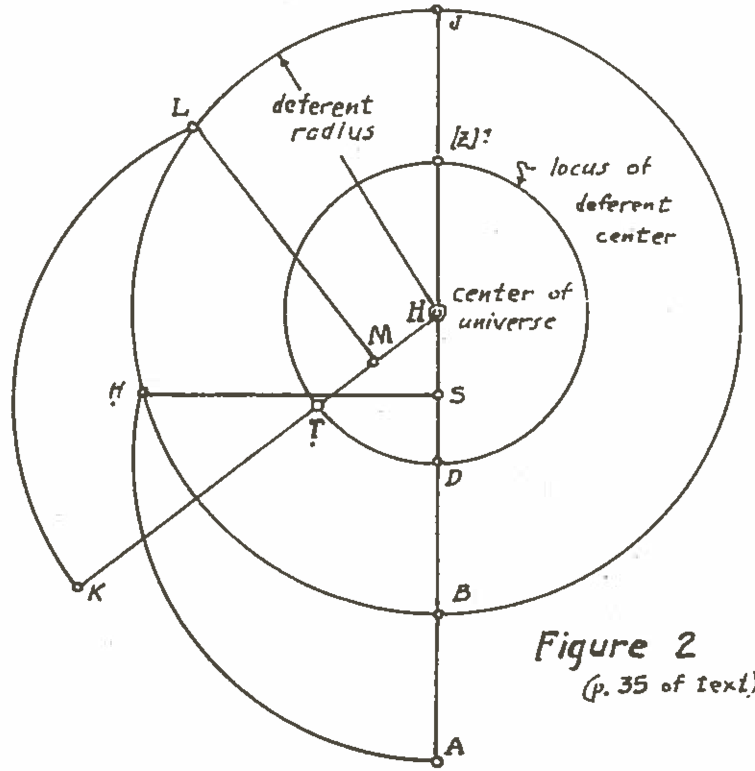


Figure 2
(p. 35 of text)

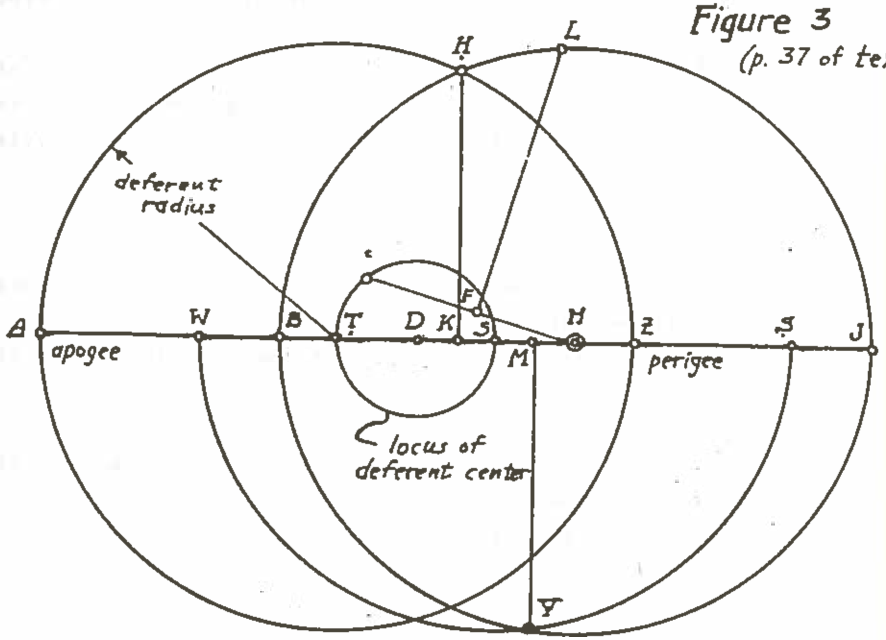
34:5 falls at the midpoint of DH. Then let the center of the deferent be at Ṭ, and we extend
6 HṬK until that becomes equal to DA, and we describe with center at Ṭ
7 and at distance ṬK a circle KL. So it will be the deferent at an (arbitrary) time, and the
8 maximum distance in it (will be) HK and the mean (distance will be) at L, and perpendicular LM falls on
9 the midpoint of H[Ṭ]. And it is apparent that the mean distance of the moon moves from
10 H to L when the apogee moves contrary (to the signs) from A to
11 K, and the center also from D to Ṭ.
12 And we assign for the determination of the mean distance in the heaven of the apogee (i.e., the deferent) of Mercury
13 point H (Figure 3) as the center of the universe and point D as the center of the circle carrying
14 the center of the heaven carrying the epicycle, and point S at the midpoint of HD, (hence)
15 the equant center. And we describe with center [D] and with radius DS
16 circle SṬ so the lines HS, SD, and DṬ will become equal.
17 And we suppose the center of the deferent (to be) at point Ṭ, which is on the prolongation of HSD, and ṬA
18 half its diameter, and with Ṭ as center and AṬ as radius we draw circle
19 AHZ which is the deferent, and with this radius also we draw with H as center

35:1 circle BHJ. So the distance of A, the apogee, from H, includes the radius
2 of the deferent and the three equal lines. And the distance of Z, the perigee,
3 from H, includes the radius of the deferent less these three lines.

35:4 So it is by necessity that the mean distance will be the radius of the deferent when
5 the excess vanishes, which is (only) exceptionally the case.

36:1 And if the center of the deferent reaches point S, the apogee will be at
2 S, and its distance from H will include the radius of the deferent except for one of the
3 three lines. And H[W], the distance of the perigee from H, is the sum of
4 the radius of the deferent and HS, when the increment is annulled by the decrease
5 which equals it, the mean distance (then) will be equal to the radius of the deferent, and indeed
6 the apogee and the perigee exchange places at this last position since point S.

Figure 3
(p. 37 of text)



36:7 which was at A became now, at Ṣ, nearer to the earth than
8 point W which was then at Z.
9 However point Ḥ, which is for the left-hand mean distance because of the
10 intersection, the perpendicular dropped from it to AZ falls on the midpoint of TḤ,
11 which is K.
12 As for point Y, which is for the intersection in the last position (<u>or</u> situation), it is
13 for the right-hand mean distance, and the perpendicular from it falls on M, which is the midpoint of HS.
14 And it is evident that the center of the deferent, if it is on (a point) other than (one of) the two points S and Ṭ,
15 it being as though it moves by its motion contrary to the succession (of the signs) until it falls on ᶜ (the letter ᶜ<u>ain</u>).
16 And we join H (to) ᶜ and bisect it at F, then erect from [F] a perpendicular
17 to Hᶜ bounded by(?) circle BḤJ, verily L will be (at)
18 the mean, right-hand distance, through which the drawn deferent passes with its (proper) radius and having (as)
19 center ᶜ.

37:1 So it has become evident as to how the left-hand mean distance is carried from Ḥ
2 to L by the carrying of the center Ṭ to ᶜ, and the difference of

38:1 the distances of its positions from the apogee of the equant, at whose center
2 is measured the constant mean motion.

38:3 Mention (<u>or</u> Explanation) of the Mean Distances of the Planets
4 In their Epicycles

5 It is apparent that the mean distance in the orbit of the epicycle will be at
6 its intersection with the deferent, if the distance is measured from its (the deferent's) center. But if it is(measured)
7 from the center of the universe, its position will vary each time. For the determination of that, let
8 ABJD (Figure 4) be the deferent with center Z, and the center of the universe H.
9 Diameter AZHJ extends in it, and we mark off on it ZṬ equal to ZH,
10 and so Ṭ will be the equant center. And we place the center of the epicycle upon
11 A, which is the apogee of the deferent.
12 It is characteristic of the epicycle that it is invariably so much smaller than its deferent that
13 it cannot enclose the earth as does the deferent, but rather is (always) away from it, and does not pass through its neighbourhood,
14 because motion through it is interdicted (as shown above). And of its (points), its epicyclic apogee is its maximum distance from the earth,
15 while its (epicyclic) perigee is its nearest point to the earth. And if we extend the radius HA
16 along its prolongation, K will be the epicyclic apogee and J its perigee.
17 Then we place the epicycle on [Ḥ],(the point) opposite the apogee, and
18 Y will then be its apogee and [F] its perigee. So when the distances are taken from Z,

39:1 the center of the deferent, their maximum will be ZK, and it is its (the deferent's) radius, increased by

39:2 the radius of the epicycle. And its minimum will be Z[J], the radius of the deferent diminished
3 by the radius of the epicycle, and one half their sum (will be) the mean distance,
4 and (this is) the radius of the deferent, without any increase or decrease. But when the distances are taken
5 from H, the center of the universe, HK will be the maximum, and it is the radius
6 of the deferent, there being added to it HZ, the amount of the eccentricity, and AK, the
7 radius of the epicycle. And the minimum will be HF, the radius of the deferent less
8 ZH, the eccentricity, and F[Ḥ], the radius of the epicycle. And half
9 their (the maximum and minimum) sum taken for the mean distance is again the radius of the deferent alone. And hence

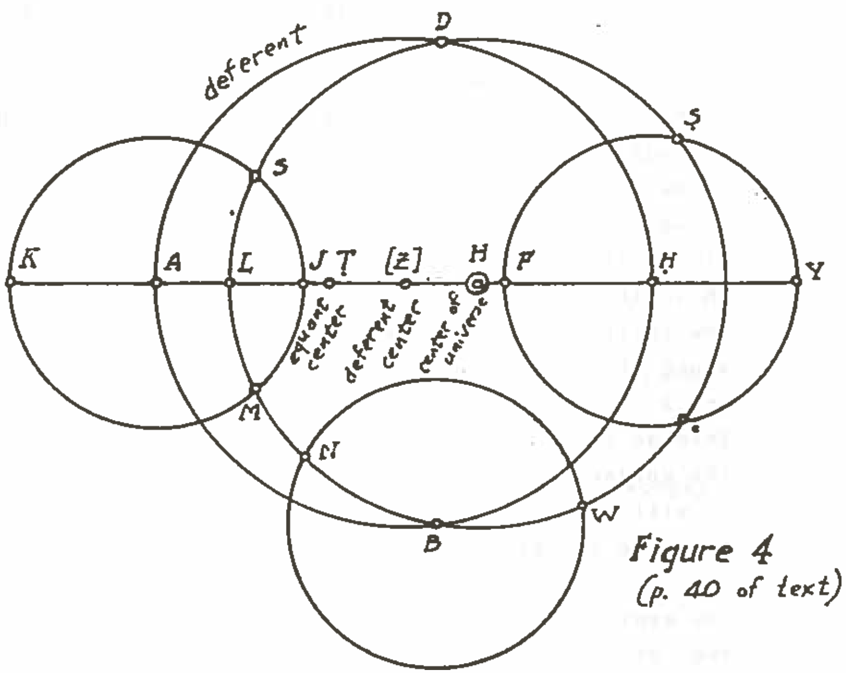


Figure 4
(p. 40 of text)

39:10 we describe with its distance as (radius), at the center of the universe (as center), circle LBD, which bounds the two mean
11 distances in the epicycle from H, the position of observation. I mean
12 that their two positions around the apogee are the two points S (and) M below the two intersections
13 of the epicycle and the deferent.
14 And at the point opposite the apogee the two points are Ṣ (and) ʿ, above
15 the (above-)mentioned intersections.

40:1 And if we put the epicycle center at B, then the two mean distances
2 at it will be the two points N (and) W. However, N is lower than the intersection (with the deferent) whereas
3 W is above it. And it is evident that the center of the epicycle, if it were at point N,
4 the right(-hand) distance would be below, and the left(-hand) distance at the intersection of the epicycle and the deferent at
5 B neither below nor above. But if it were at point W, so that (the epicycle)

41:1 passed through B, all would be opposite to what we have mentioned, I mean that the left one
2 will be above while the right(-hand) one would be on the same (above-)mentioned intersection. (Now,) for the determination of the distance of the intersection
3 from the epicyclic apogee, we turn from this figure to what we need, (Figure 5) and we extend AD tangent
4 to the deferent at A, and ZḤT tangent to the epicycle at Ḥ, and it is known that (arc)
5 ȦT is the greatest of all the equations due to the epicycle by its magnitude, angle AZṬ.
6 But the triangles ʿAZ, AḤZ, and ʿḤA are similar, and so angles ʿAḤ (and)

41:7 AZ[Ṭ] are equal, so arc DḤ has the magnitude of the maximum equation in the
8 heaven of the epicycle. But B, the position of mean distance, is not at the midpoint
9 of arc DḤ. Let us drop perpendiculars ḤS and BM to AZ, and join
10 B (to) A, B (to) S, and B (to) [J]. Then from the similarity of triangles AḤZ, ASḤ, and SḤZ,
11 the product of ZA and SA will be equal to the square of AḤ, and the product of JA
12 which is twice AZ, and AM, (which is) half AS, is therefore equal to the square of AḤ,
13 which is equal to AB. So the ratio of [J]A to AB is as the ratio of BA
14 to AM. And so the two triangles [B]AM (and) [J]AB are similar. But triangle [J]AB

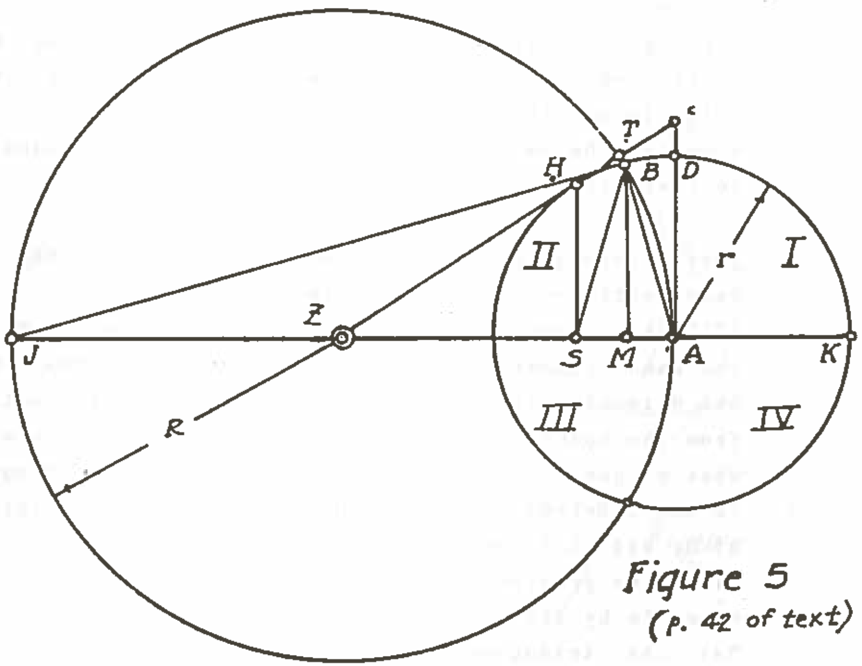


Figure 5
(p. 42 of text)

41:15 is (inscribed) in a semicircle, and so angle AB[J] is a right angle, and angle AMB is equal to it,
16 and it also (is) a right angle. And MB is perpendicular to AZ, and the ratio of ZA to
17 AB, I mean AḤ, is as the ratio of BA to A[S]. So triangles ZAB and
18 BAS are similar. But triangle ZAB is isosceles (with) legs AZ (and)

42:1 ZB. So triangle BAS is also isosceles, with legs AB (and) BS. And M
2 is the midpoint of its base, and so MB is its altitude. And since M is the midpoint of AS,
3 AB will not be the bisector of arc DḤ, the maximum equation, as
4 was evident in the subtending of the sines, and the epicycle equals, in this respect, the deferent.

43:1 And that is, if the sine of the maximum equation is taken it will be AS,
2 and its half, AM, and the arc of this half is DB. And if (DB) is added to KD,
3 the quadrant, there will result KDB, the first mean sector, because
4 it is measured to the center of the deferent.
5 And it is to this that Abū Maʿshar has referred in his zīj and said: "As for the determination of the mean distance
6 "in the epicycle, we multiply the sine of the epicycle radius of the planet by
7 "itself and divide by twice the total sine, and we determine the arc (sine) corresponding to the result, and it is added to three
8 "signs and there results the distance of its mean distance from the epicyclic apogee."
9 And it is as if he means that the epicycle radius is arc AB.

43:10 The ratio of A[J], twice the total sine, to BM, its (i.e., angle BZM's) sine, is not equal to the ratio of

11 BM to MA. But the ratio of A[J] to the chord AB is as the ratio

12 of the chord AB to AM. And AM is also what gives (the thing) sought by him.

13 For verily arc AB, if he takes it as the maximum equation, it is not

14 it. The maximum equation is rather (arc) AṬ, and even if the radius of the epicycle

15 was known to him, we must use it as it is.

16 But as to the measurement with respect to the center of the universe, we suppose that the center of the epicycle (Figure 6)

17 is at A, the deferent apogee, and let its intersection with the circle bounding the two mean distances be B.

18 And we join B (and) H, and it will be equal to Z[A]. And AB,

44:1 the radius of the epicycle, is known, and ZH is known, so triangle ABH is known as to sides.

2 So the normal [BH] is known.

3 And if we transform it(s) magnitude to the scale by which AB is the total sine,

4 and we then determine its arc sine, arc BM would be determined, and DB, which is required

5 is its complement, hence it is (now) known. So $\widehat{KB}$, the distance of the apparent mean distance, it being

6 the first adjusted sector, would be known. But the distance of point ᶜ from the epicyclic apogee is known,

7 and ᶜB, the mean depression, is known. In like manner we put

8 the center of the epicycle at J, the point opposite the apogee, and let it intersect with the circle

9 bounding the two mean distances at S and join S (to) H. Then triangle HS[J]

10 will have its sides known, and SL which is its normal, will be

11 known.

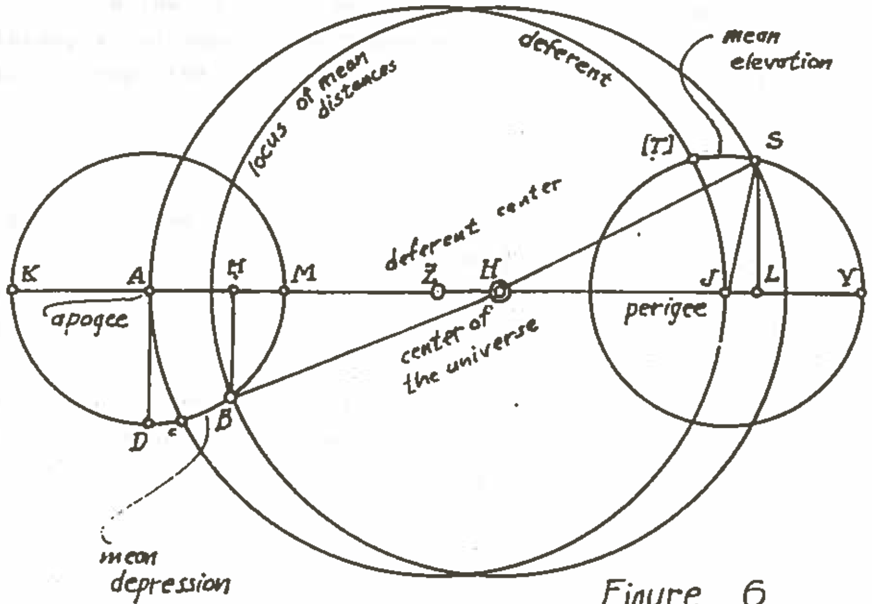


Figure 6
(p. 45 in text)

44:12 And if its magnitude is converted to the scale by which SJ is the total sine, and

13 its arc sine is found, $\widehat{YS}$ will become known, and it is the magnitude of the first adjusted sector.

14 But the distance of point S from the epicyclic apogee is known, so SṬ,

15 the mean elevation, is known. And verily we have called them the two means because of their equality on the

16 right and the left and the inevitable inequality of any others than they.

45:1 As for positions other than these two, let the center of the epicycle be at B (Figure 7),

45:2 and extend from T, which is the center of the equant, TF[B]ʿ.
3 and ʿ will be the mean epicyclic apogee, from which is (measured) the beginning of the anomaly, which
4 is called also the non-modified apicyclic argument (a flaw in the ms?), its beginning (at ʿ?), and F is the mean
5 epicyclic perigee.

46:1 And we draw from the center of the universe HSK, and K will be the [apparent]
2 epicyclic apogee from which is the start of the anomaly, or the argument, or the corrected epicycle. And S is
3 its apparent perigee. Let the intersection of the epicycle with the (above-)mentioned circle be point
4 D, and drop DM perpendicular to HK, and HL [and ZH] perpendicular to TB.

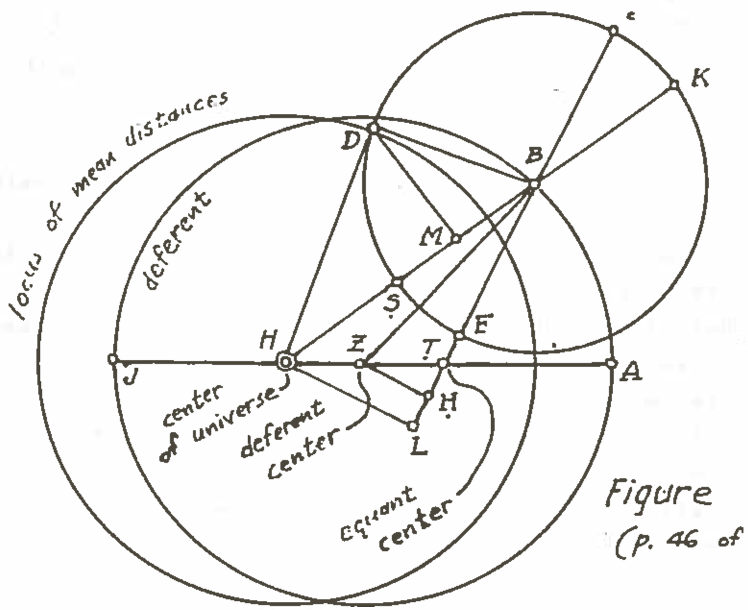


Figure 7
(p. 46 of text)

47:1 So angle ATB will amount to the unmodified (i.e.,mean) longitude, and it is also called the center.
2 And hence triangle HLT has its angles known, and in it HT is known, so its sides
3 are also known. And ZH is half HL, and so H is the midpoint of LT.
4 And ZB is known, so HB is known. And all of LB is known so HB, which
5 is the hypotenuse of a right triangle formed by it and HL, is known, and HD is equal to ZB. So triangle
6 HDB has its sides known and its normal, I mean DM, is known, and after
7 transformation (of units), it will become the sine of the arc SD, and KD is its supplement.
8 It is the first modified sector, now determined. And because the depression of the intersection of the epicycle
9 with the circle bounding the two mean distances at the time when the center of the epicycle
10 is at the apogee is the mean depression, and its elevation at the time when it is on the (point) opposite the apogee
11 is the mean elevation, they will not be their extreme (values), which
12 are (the ones) sought after in practise.
13 And so we place the center of the epicycle so that its circumference will pass through point L (Figure 8)
14 on that circle, so that this point will be the position of the two mean distances
15 at the extreme of their depression. And the other two will then be a little bit above, and that is a characteristic of the two
16 positions on the sides of the apogee if the center is on them, but that (position)
17 which is before the apogee L will be the left mean distance at it, and that (position) which

47:18 is after it will have L at it as the right mean distance. And let us take one of them as an example;

48:1 then the other can be imagined. And let us place the center at B, so that L becomes

2 the right mean distance, and produce BD tangent to the deferent, and drop

3 LḤ perpendicular to BZ. And because BZ is the radius of the deferent,

4 and ZL is the difference between it and AL, which (difference) is equal to ZH, the distance between

5 the two centers (i.e., the eccentricity), and BL (is) the radius of the epicycle. (So) triangle BLZ,

6 whose sides are known, will be known, and (also) LḤ, the normal, and ZH and ḤB,

7 the two parts of its base, and (likewise) BḤ, the sine of arc DL. So if it(s length) is transformed to the

8 magnitude (i.e., scale) by which BL is the total sine, arc DL will be known.

9 And it is the extreme of what there is for the mean distance, insofar as depression from the quadrant (point) is concerned. But the point

10 of intersection of the epicycle with the deferent is known, and so the depression from it is known, and let us call it the

11 total depression. It (the depression?) will be non-existent when the center is at F, and

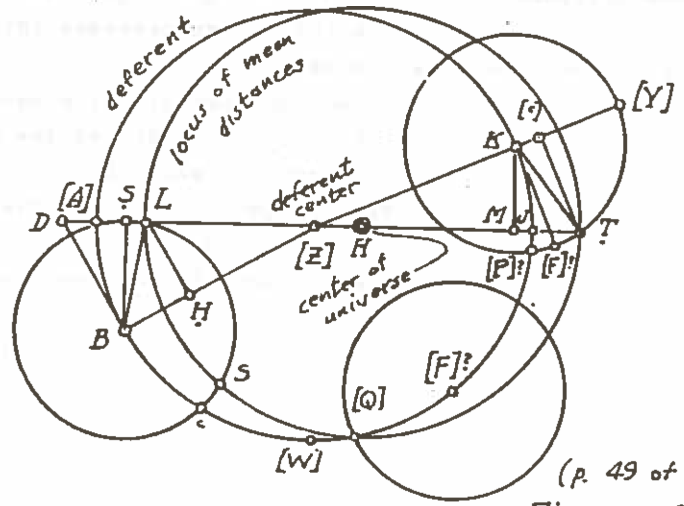
12 this mean distance will result at the same time, at the same intersection which is Q.

13 The left one will then be above the deferent. However, the depression of the mean distance, which is

14 S^{c}, will not exist when the center is at W, so that

15 the left one will be at the node and the extreme of its magnitude will be when the center is at a point before the apogee

16 by the amount of arc AB.



(p. 49 of text)

Figure 8

49:1 And this (also) is the situation for the extreme (value) of the elevation of the two mean distances at two positions

2 on the two sides of the point opposite the apogee. Let one of them also be point K, and

3 at it the center of the epicycle, and let its circumference pass through point Ṭ, so it will be the right (hand) distance.

4 and we drop KM perpendicular to ZṬ and Ṭ^{c} perpendicular to ZY.

5 So, since KZ is the radius of the deferent, and ZṬ (is) the result of adding it to JṬ,

6 which is equal to ZH, triangle ZKṬ will have its sides known, and the ratio

7 of its normal to KZ (is) as the ratio of ^{c}Ṭ to ṬZ. And after transformation (of scale)

8 arc ṬY will be known, and its complement [plus PF?] is the total elevation, and (it is so)

50:1 because of the center being at [K?], since the right mean distance

2 will then be at the point of intersection (P?), but the arc between it and F is

3 twice the arc (sine) of one quarter of the diameter of the epicycle. Let it be called the arc of the chord. However, (as for) the distance of point B

4 from the apogee, verily its sine is ṢB. And its ratio to BZ, the radius of

5 the deferent, is as the ratio of LḤ, the normal of triangle ZLB, to LZ, the difference

6 between the radius of the deferent and the distance between the two centers, I mean ZH. So arc

7 AB is known; let it be called the arc of the distance of the extremity.

8 And it resembles the operations by which we transform the equation of the epicycle in the zījes from

9 the quantity calculated at mean distance to what it is required for it at each distance; we transfer

10 this total elevation and total depression to their two magnitudes at both sides, the right to

11 the left for each distance, if one imagines before him the beginnings of the distance sectors and the

12 beginning of the elevation and the depression and their vanishing. That is that the beginning of the first sector is (at) the apogee.

13 That of the second is distant from the apogee by the magnitude of the first sector, and the beginning of the third is (at the point) opposite

14 the apogee. The beginning of the fourth is the complement of the first with respect to a revolution, I mean before the apogee

15 by the magnitude of the first sector.

50:16 As to the beginning of the depression, it is near the beginning of the fourth sector. Moreover, the left mean distance

17 is before it by the amount of the arc of the chord, and the right one thus is after it to the amount of its magnitude.

18 And it vanishes opposite the two beginnings, I mean that the vanishing of the left one is before the beginning of the

19 second sector by the amount of the arc of the chord. And the vanishing of the right one is after it. The extreme (value) of the depression (is) around the apogee

51:1 by the (amount of the) arc of the distance of its left (-hand) extreme (value) before it, and the right(-hand value) after it. The case of the elevation resembles

2 that of the depression, but its beginning is near the beginning of the second sector, and thus the arc of the chord

3 for the left (one) is before it, and for the right (one) after it. However its vanishing is near the beginning of the fourth sector to (the amount of) the arc

4 of the chord, for the left one before it, and for the right one after it. And the extreme (value) of the elevation occurs near the point opposite the apogee, at

5 two positions distant from it by (the amount of the) arc of the distance of the extreme elevation, for the left one before it, and for the right one

6 after it. So when the position of the epicycle center is known, its situation will be known

7 with respect to these limits which we have enumerated, and there is no doubt but that our objective will be achieved

51:8 However (in dealing with) altitude as well as with depression, we use for each (one) its extreme, without the other extreme,
9 and the side, left or right, will also be assumed. So our objective
10 (is) its beginning if it is increasing, and the position of its end if it is decreasing,
11 without mixing one with the other.
12 And if the center lies between the positions of the beginning and its extreme we take out
13 of the total (i.e. extreme) an amount equal to the distance of the center from the beginning by multiplying the distance from the center to
14 the intended beginning by its total (i.e. extreme value), and we divide what results by the distance of its extreme position
15 from its beginning. So the required depression or elevation results, on the
16 intended side. And when the depression or elevation is ascertained for the assumed time, the
17 position of the mean distance on the epicycle for that position on the deferent
18 will become known.
19 And what we need in these operations is the eccentricity, I mean

52:1 the sine of the equation due to the deferent. And what has already been mentioned as to maximum equations
2 is what is due to the equant, and if you take its two parts and then take half
3 there would result (the distance) between the center of the universe and the deferent centers, because they are at the midpoints
4 of the segments between it and the equant centers.
5 And what is in the Almagest regarding that is: for Saturn three parts
6 and thirty-four minutes, and for Jupiter two parts and forty-one minutes and a half,

52:7 and for Mars six parts and thirty-three minutes and a half,
8 and for Venus one part and fifteen minutes, and for Mercury at the least magnitude three parts,
9 and for the greatest nine parts.
10 As for the first (i.e., least distance) it is when the center of the deferent, on the circle carrying it, is
11 on the equant itself. Let it be Ṭ (Figure 9), and the center of the universe H. But
12 the greatest (distance, it occurs) when the [apogees] of the deferent and the equant unite so that
13 the center of the deferent circle will be at K, which is on the extension of HṬD.
14 As for the rest of the time, let (the deferent center) be, for example, at Z. Then HZ will be (the distance) between
15 the center of the universe and the deferent and it is what is required, without HṬ which is the sine of
16 the maximum equation. And arc ZK is equal to the distance of the center of the epicycle from
17 the apogee along the direction of the succession (of the signs.) So its sine, ZḤ, and its cosine, ḤD,

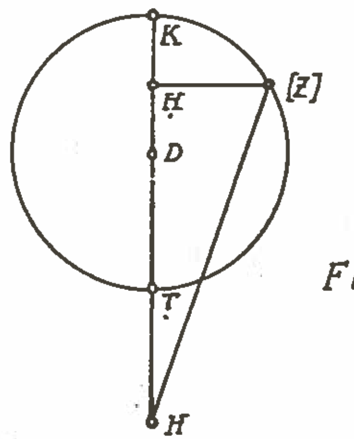


Figure 9 (p. 53 of text)

52:18 are known. And if they are transformed into (units of)
which [D]K is three parts, and ḤD is added
19 to DH, which is six parts, and there is decreased from
it according to the position of Ḥ

53:1 from the center D there will result ḤH. The hypotenuse
of the right triangle having it and ZḤ as legs
2 is the desired (object).
3 But the radii of the epicycles according to what
is in the Almagest are:
4 six parts and a half for Saturn, eleven parts and a
half for Jupiter, and for Mars
5 thirty-nine and a half parts, and forty-three and one
sixth parts for Venus,
6 and twenty parts and a half for Mercury, and the
magnitudes of the maximum equations which are due to
the epicycles will follow them (accordingly).
7 The moderns have followed in it Theon of
Alexandria, and in the Canon it is,
9 for Saturn six parts and thirteen minutes, for Jupiter
eleven parts
10 and three minutes, for Mars forty-one parts and nine
minutes, and for Venus

54:1 forty-five parts and fifty-nine minutes, and for
Mercury twenty-two parts and
2 two minutes, and it is thus in the Almagest.
3 But in Ibn al-A‛lam's zīj it is for Saturn
diminished by twenty-five minutes,
4 and for Venus increased by nine minutes, and for
Mercury increased by twenty minutes.
5 In the Shāh Zīj it is five parts and forty-four
minutes for Saturn,
6 and it may be in some copies less by eight seconds
(sic) and in some others by one minute.
7 For Jupiter, ten parts and fifty-two minutes; but with
Abū Ma‛shar

54:8 it is less by eight seconds. For Mars forty-one parts
and thirty minutes,
9 and it is found in some of them diminished by one
minute, and with Abū Ma‛shar it is as in the Canon,
10 and by increasing five seconds. For Venus, forty-
seven parts and eleven minutes,
11 and it may be diminished by one minute in some of the
copies. For Mercury, twenty-one parts
12 and thirty minutes, and it may be diminished by about
half a minute in some of the copies.
13 And with Abū Ma‛shar it is as in the Canon. But
al-Fazārī and al-Khwārizmī have
14 them like what is in the Shāh Zīj, since it is the
Hindu way.
15 And it must be that Ya‛qūb ibn Ṭāriq is in
agreement with the two of them, but what is
16 in his zīj for Jupiter is decreased by twenty-two
minutes, and for Venus decreased by
17 fifty-five minutes.
18 And al-Sarakhsī has followed in the case of
Saturn the Shāh Zīj and in the remaining one the Canon.
19 However, Paulus put the maximum equations (as) the
circumferences of the carrying epicycles

55:1 by multiplying the equations by three hundred and
sixty and dividing the result by the total sine,
2 which, according to him is fifty-seven parts and
eighteen minutes. But the equation
3 of Saturn is six parts and twenty-two minutes, and the
circumference of its epicycle (is) forty;
4 and the equation of Jupiter is eleven parts and thirty-
two minutes and the circumference
5 of its epicycle (is) seventy-two; and the equation of
Mars (is) forty parts and
6 thirty-two minutes and the circumference of its epi-
cycle (is) two hundred and fifty-five; and the equa-
tion of

55:7 Venus is forty-five parts and fifteen minutes, and the circumference of its epicycle
8 (is) two hundred and ninety; and the equation of Mercury (is) twenty-one parts and thirty-six
9 minutes, and the circumference (Here a passage is
10 repeated in the text.) of its epicycle (is) one hundred and
11 thirty-five.
12 However, in their other zījes their sayings are not stable, and they can not be relied upon, and that is why
13 I have shunned talking about them.

14 Mention (*or* Explanation) of the Sectors
15 in both Heavens According to the Well-known Opinion

16 The differences found both in the deferent and
17 the epicycle are of two kinds (each), one is the distances included between two extremes in greatness and smallness
18 and a mean between them. And it has been shown above that by them the two orbits are divided into
19 four sectors.

56:1 But the second type (concerns) the differences in the motion due to the difference
2 between the two centers (i.e. eccentricity). For the motion near the apogee (is) at the extreme of slowness, and near its opposite (is)
3 at the extreme of rapidity. But at two points between them, at the extremities of the chord which is perpendicular, at the
4 center of the universe, to the diameter passing through the apogee and its opposite, it (the angular velocity) is at its mean,
5 equal condition. And at them will be the maximum equation, as it vanishes essentially at

56:6 the apogee and its opposite, because of the coincidence of the lines coming out to them from the two centers.
7 And by these four points the deferent is divided into four sectors,
8 and the drawing (Figure 10) shows that and helps to understand it.
9 Let ABJD be the deferent with center Z which is external
10 to H, the center of the universe, and we extend through it the diameter passing through the two centers.
11 So A will be its apogee, and J the opposite of the apogee, and they are the (respective) beginnings of the first and third sectors.
12 And as for the beginnings of the second and fourth sectors according to this last opinion
13 which we are considering, let us pass chord BD through its center, normal
14 to the diameter AJ. So the two points B (and) D will be the (beginnings of the above-)mentioned sectors,
15 so that the variations in the equation will be at the four points A, B, J, (and) D.
16 However, at the two points A (and) J the equation will vanish essentially because of the coincidence of the two lines
17 issuing from Z (and) H. Then its excess will be great at them. As for (the situation)
18 at the rest of the points, the two (above-)mentioned lines will be distinct, and they will bound the angle of the
19 equation, like angle ZBH, and it is the greatest of all angles of the equation which precede it

57:1 or which come after it, I mean the preceding (ones) like the one at Ḥ. And for this drop ZK perpendicular
2 to HḤ. And HZ will be the hypotenuse of a right triangle having as legs ḤK (and) ZK.

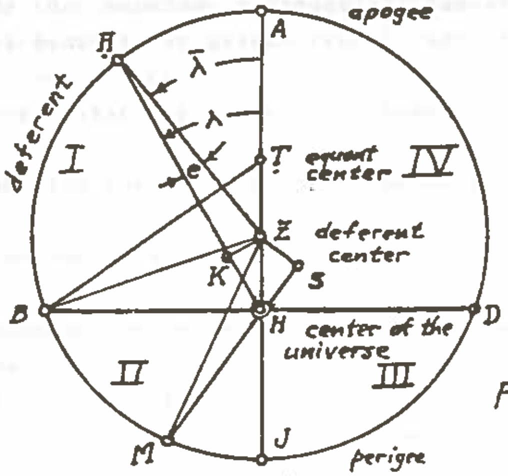


Figure 10
(p. 57 in text)

57:3 hence ZH will be greater than ZK. But they are chords
of the circles circumscribed about
4 the two triangles ZKH (and) [Z]HB, which are right
angled, and they are equal to circle ABJD
5 because the radius of the three is one magnitude. So
ZH (is)
6 the chord of an arc greater than the arc of chord ZK,
and angle ZBH is greater
7 than angle ZHH.
8 And I mean by the ones that come after, (those)
like the one at point M. For it, drop ZS perpendicular
9 to MH, and (the argument is) as what has preceded in
comparing ZS with ZH so as to make clear

58:1 that ZH is greater than ZS. So angle ZBH will be
2 greater than angle ZMB, and the position B (is) for
the variation in the equation, because
3 at it it is at the greatest of its magnitudes, and its
situation differs around it. And the equations start
4 at A, increasing up to it, and they stop (increasing)
at its greatest magnitude. Then it returns
5 from it to diminishing boundaries. And thus is also
the situation at point J. So if this

58:6 orbit were carrying the epicycle of one of the planets,
point T would be
7 the center of the equant, and the greatest of the
equations will be at point B.
8 Nothing changed in it except the magnitude because of
the transformation of HZ into HT. And in like manner
9 the deferent at point D for (any) one of the equations
from J up to it (is) increasing,
10 and from it to A is decreasing.
11 And for determining (something) like that in the
epicycle, let KLD (Figure 11) be the epicycle
12 and its center B on its deferent, and let us produce
to it from the center(s) of the deferent
13 and the universe two lines determining the two epi-
cyclic apogees, K (and) ᶜ, and produce from Z radius
14 ZDH tangent to the epicycle at D, which is the position
of the maximum equation.
15 Because the lines extending to (points) other than
point D and its counterpart on the other side
16 lie between the two like lines. And so angle BZ[D] is
greater than
17 any angle bounding it. BZ is one of these lines, and
the radius

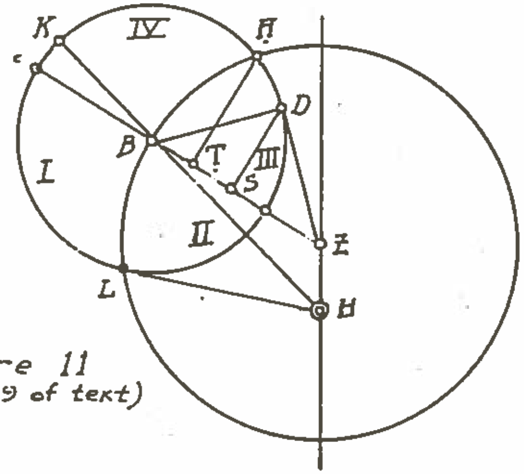


Figure 11
(p. 59 of text)

58:18 of the epicycle will be the sine of this angle, and it is the normal HT
19 to BZ, because each one from it and from BD (is) normal from one of the ends

59:1 of the arc to the diameter (or hypotenuse) emanating from its other end. And we have already mentioned
2 that the normal DS represents in the epicycle the chord which bounds in
3 the deferent the two positions of the extreme equation. And point S stands for
4 the center of the universe. I mean that the ratio of SB to D[B] is as the ratio of (the distance) between
5 the two centers in the deferent (i.e., the eccentricity) to its radius. And so arc KD is equal
6 to the first mean sector which does not change. However, we put it on this side
7 so as not to complicate the figure by our drawing of HL tangent to the epicycle, and arc
8 K̇L is the adjusted time (i.e., velocity) sector. The adjusted anomaly is measured with respect to it
9 because its starting point is from the epicyclic apogee K. And the first and fourth sectors in both orbits (meaning the epicyclic and the apogee sectors)
10 are called the ascending (ones) and the remaining the descending.

60:1 And that is either by measuring their centers with respect to the center of the universe, for verily each of
2 the two of them is higher (ṣāʿid), than the reality of lowness, and it has [raised] the two sectors with it, and the remaining two
3 stay lower than it. However, as to their being above the mean distance, (hence) the other two will be
4 below it. Whereas the planet will be descending in the first, the ascending (or higher) sector,

60:5 and in the second, the descending one, it will be descending, and in the third, the descending one, it will be ascending, and in the fourth, the
6 ascending (or higher) one it will be ascending.
7 And what makes the first method, in which the orbit has been divided by
8 the two mean distance (positions), preferable to this second one in which it was divided by the two positions of the maximum equation,
9 is that the equation is what gives the travel its increase or decrease of speed. For the speed,
10 if it were free, not borne on an orbit bounding it, then it would not be limited,
11 for it has the potentiality of increase. Everything that is potentially increasing starts
12 from its smallest (value), before which it had been null. Then the opposite occurs, by which
13 the speed is slowed down and goes back to its initial value. This is (known as) deceleration.
14 But the deceleration is bounded, because the initial value is the least it can assume. And when
15 the motion is from the epicyclic apogee in a direction opposite to the succeession (of the signs), like that of the moon according to Ptolemy,
16 the slowing down for it would be as it is at the apogee in its (the apogee's) orbit (i.e. the deferent). But
17 the motion of (any one of) the five planets in its epicycle will be from the epicyclic apogee
18 along the succession (of the signs), equal to the motion of its center. Hence its (maximum) speed occurs at the epicyclic apogee,
19 and its slowing down at the epicyclic perigee. And obviously, between the two positions of the maximum equation

61:1 in the inferior segment (are) the two essential stationary points in the path. At one of them

61:2 starts the acceleration which ends in increasing to its extreme at the epicyclic apogee, to the
3 other which ends in deceleration.
4 And the situation between them along the lower side differs from that between them along
5 the upper side (as the) difference between existence and nonexistence, for it is going back in appearance, contrarywise,
6 and in addition to that, the speed begins increasing at one, and stops decreasing at (the end of)
7 the other, (just) as forward motion between them has followed (the retrogradation). But if the orbit were to be divided into sectors according
8 to the travel and what it requires, what is the objection to dividing it by the two stationary points, so that
9 the first sector will be from the middle of the forward motion to the first station, and the second
10 from the first station to the midpoint of the retrogradation, and the third from the midpoint of the retrogradation to the
11 second station, and the fourth from the second station to the middle of the forward motion. There is no objection to that
12 excepting the claim (of some) that a certain effect is accounted for in the previous (alternative), but not in the latter, such as the ebb(?) and flow of the tides, but that is rather far-fetched.
13 But in such cases retrogradation and forward motion should be given precedence in the explanation, including
14 the change in the equation from increasing to decreasing, unless it is claimed of an effect which is foreign to
15 the consistent laws of nature in the craft of astrology. But no one dares
16 claim something like that unless he is short-sighted and bound to fail.

61:17 And this meaning will become clearer when we mention the transit in thickness; verily
18 it really appertains to the first method rather than the second.
19 And it is necessary after what we have stated to explain both methods

62:1 from a practical point of view for those who want to use them, and it is necessary for that to utilize the quantities
2 which are found between the centers (i.e. the eccentricities) and the diameters of the epicycles. And nothing will be listened to except
3 a temperament unbiased by the germ of fanaticism, and the taint of insistence, and the lust for victory
4 in utilizing any of these, except what is clearly apparent, or accompanied by the best of proofs.
5 And this is the case of the talented Ptolemy; his works are to the works of others as
6 wakedness is to sleep, and his position is (actual) sight as compared to the hallucinations of dreams. And if
7 time has not helped us thus far to consider cases other than that of the sun, we
8 use what is in the Almagest concerning that and say:
9 As for the apogee sectors, the first of them, according to the first opinion, in the case of the sun (is)
10 to make the maximum equation a sine and it will be the eccentricity.
11 The arc (sine) of half of it is to be taken and added to ninety, and the sum will be the first sector. Its complement with respect to the circumference,
12 which is three hundred and sixty, is the beginning of the fourth sector, and we need not stop to explain the third
13 since its beginning is always from the midpoint of the circumference. To these sectors is measured

62:14 the unmodified argument of the sun.
15 However, according to the second opinion, the maximum equation is to be added to ninety,
16 and it will be the magnitude of the first sector, and the maximum equation is to be subtracted
17 from two hundred and seventy, and there remains the beginning of the fourth, and to it also is measured the argument which is not
18 modified (i.e. the mean argument).
19 As for the law of the adjusted argument, if it is desired to measure (the sectors) by it, (the beginning of the second sector is at) the complete quadrant,

63:1 and the (beginning of the fourth sector is at) three complete quadrants without increase or decrease if it is (for)
2 the mean motion (sectors). And its equations are put in the zīj without
3 its author having the kindness to explain the operation or to generalize it; and among the authors of zījes are those who find
4 in the elements of the motions a reason for putting them (the sectors) in the tables of the equations, and they
5 return to them upon completing the operation with the equation. However, before that there is no consideration for them
6 except a partial consideration, characterized in each zīj by separate numbers.
7 And these are like Ḥabash al-Ḥāsib (i.e. the Computer) in the operations for the moon, and like Abū al-Faḍl ibn Māshāllāh
8 in his summary of al-Khwārizmī's zīj, and Ḥabash's zīj, and like Kūshyār ibn Labbān
9 in his Jāmiʿ Zīj, and like Abū al-ʿAbbās al-Ḥawālfaʿsī in his summary
10 of al-Battānī's zīj. And like this is the situation regarding the sectors of the planets in their

63:11 deferents if what is used for them are their maximum equations caused by the
12 equant. But mention of their magnitudes has already been made, and by them their unmodified (i.e. mean) longitudes and centers are measured,
13 whereas the measuring of the modified (ones) is at the complete quadrants.
14 As for the moon, in which nothing but epicycle sectors are used, whoever wants
15 them (i.e. sectors) in its deferent must consider them by its double elongation, and that is because the center
16 of the epicycle of the moon will meet the apogee both at conjunctions and oppositions, whereas it meets
17 the opposite (point) to the apogee in the two quadratures.
18 But as for the epicycle sectors, if they are considered with their equations, in the same way as
19 for the apogee (sectors) their magnitudes will result approximately according to both opinions.

64:1 So, according to the first opinion, the anomaly is considered, adjusted by half the equation
2 of the center to be measured up to an epicyclic apogee required by the deferent between
3 the mean (epicyclic apogee), which is demanded by the equant, and the apparent (epicyclic apogee, determined) from the center of the universe,
4 resulting from the whole equation of the center.
5 Whereas according to the other opinion, the true (i.e., adjusted) anomaly is considered with the whole equation of
6 the center, and that is because deep investigation into it is long and its methods are indicated in what has preceded.
7 And there is a book by al-Ḥasan ibn ʿAlī ibn ʿAbdūs, "Introduction to the Profession

64:8 of Astrology" (Al-mudkhal ilā ṣanāʿat al-aḥkām). It
was mentioned in it, and this is his talk.
9 Most of the authors who discussed sectors have
erred in most
10 of the chapters on it, or forgot it, especially in the
case of the moon, because of the complexity of its
motions. And some
11 of the moderns of Baghdād aspired to discuss it and
made in (the case of) the moon
12 a very serious error which was not detected by any of
the people of their time, for they mentioned that the
moon at the time
13 of conjunction will be at the epicyclic apogee. But
they make a mistake there, for indeed its epicycle
14 center will be at the apogee of its eccentric orbit at
that time by (virtue of) its mean
15 motion.
16 However, the moon (itself) at the time of conjunc-
tion will be in all positions
17 on its epicycle. And he said that there are four
chords for the moon in its epicycle. They have
18 halves, so they become eight, and because it travels
its epicycle twice every month
19 by doubling these chords they become sixteen. But he
was mistaken about it, since the moon travels

65:1 its epicycle in twenty-seven days and thirteen hours
and one third, whereas
2 the center of its epicycle traverses its eccentric
orbit twice every month.
3 And then he used in ascertaining the parts of the
sectors of its epicycle, the second compound equation
4 which occurs according to its elongations from the sun,
instead of the first single equation,
5 which it obeys in the motion of the difference. And
he made a mistake in the magnitudes
6 of the sectors of the sun, for he made the first and
the fourth smaller than the second and the third.

65:7 We (ourselves) have not happened upon the book of
the (above-)mentioned (person), but the account (about)
him, if
8 it is true and if it was not due to jealousy and anger,
it indicates in his case
9 a non-studious listener, and this is the case with
most of the class of the astrologers; they babble
proudly
10 about things they barely hear, without verifying them,
and they are satisfied by associating fancies with
them.
11 And [taking] the midpoints of the chords to get the
sixteen is silly, and it sounds as if what was inten-
ded by it
12 was the coupling of the four distances in the epicycle,
I mean the farthest and the nearest
13 and the two mean ones, with the four in the deferent.
There will be sixteen (couples). But
14 by the equality of the two mean ones it becomes nine,
and moreover, if in the epicycle there were
15 eight, it would not become so by repetition of the
rotation sixteen times, whether the rotation were for
the moon
16 or the center of the epicycle. So there is no objec-
tion to making it thirty-two in two months
17 and doubling them twice. And even though bisecting
those chords was
18 because of the traces of al-baḥārain(?) the midpoints
of the quartiles(?) of the deferent are more wanted.
19 And of what has been said concerning the defi-
ciencies of the two superior sectors (as compared) with
the inferior ones,

66:1 I have no justification for it except that the deferent
was divided into equal quadrants at
2 points A, B, J, (and) D, of which A is the apogee.
Then their true longitudes are taken, so that the
magnitude

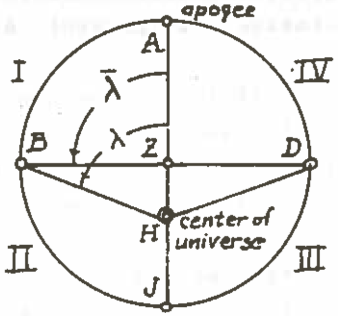


Figure 12

3 of the first sector became that of angle AHB, and it is less than a right angle
4 because it is (an angle) opposite to angle AZB in the interior of the triangle, and similarly for angle
5 AHD, by which is seen sector A[D], the fourth (one). And sector BJ will become
6 the second [seen by] angle BHJ, external to triangle HZB.
7 And like it is sector [J]D, the third, [seen by] angle JHD, and that
8 is what we wanted to show.

9 Explanation of the Increases
10 and Decreases by which the Planets are Described

11 In the case of the apogee sectors which are made dependent upon the distances, according to the first opinion,
12 there follow for the planets the problems of nearness and farness with respect to visual perception.

67:1 The planets at their apogees are seen (to be) less in size and lacking in light, and at

67:2 the (points) opposite the apogees larger in size and richer in light. And by necessity at the two
3 mean distances they will be in a mean and average situation as to them (i.e. apparent size and light) from them (i.e. the extreme positions). Then
4 in the first and second sectors it will be increasing in light and magnitude because of its descent and the increase
5 in its nearness. But in the third and fourth sectors (it will be) decreasing in them (i.e., these two qualities) because of its ascent and the decrease
6 in its nearness. And this is following the example of those who call the moon waxing in light from (first) crescent to
7 opposition, waning in light from opposition to the (last) crescent.
8 But he who thinks that it is deficient in light in the half which has the
9 conjunction at its center and is surrounded by the two quadratures, and excessive in light in the half having
10 the opposition at its middle, he considers for it the equality of light and darkness in what he perceives of its body, that being
11 at the two quadratures, its like in the (case of the) planets is to be, in the first and fourth sectors,
12 deficient in light and size, that is, from the normal magnitude, and in the
13 second and third sectors excessive in them, that is, from that magnitude.
14 But in (the case of) the apogee sectors which are set up on the basis of motion and of the magnitude
15 of the equation, it undergoes what it did in the first concerning light and size,
16 but approximately. For their beginnings are not coincident with the mean distances, and they undergo
17 in them also other increases and decreases, and they are of (various) types. Some of them are of the type of travel, since it is slow(est)

67:18 in the apogee, and at its opposite (point it is) fast(est), and at the beginning of the even sectors
19 (is) average. So it therefore ranges in the first from slow(est) to the mean,

68:1 and in the second it ranges from the mean to the (maximum) speed, and in the [third] from (maximum) speed
2 to the mean, and in the fourth from the mean to slow(est). And of them (the types) is the equation,
3 which is increasing in the odd sectors ranging from little to much, and in
4 the even sectors diminishing, ranging from much to little, that is, in the epicycle.
5 And the case of the equation in it is like it (in the deferent), I mean, it is increased in the two odd ones and diminished
6 in the two even ones, and from it is the computation which is, in the first and second, diminished because the
7 true longitude then is less than the mean, hence (there is) the necessity of decreasing the equation, and in the third and fourth
8 (it is) increased because then the true longitude exceeds the mean, hence (there is) the necessity of the increase of the equation. And of them (i.e. the types)
9 is the number which, in the first and second, is increased in it, and in the remainder deficient. And this was
10 because of the two rows (<u>or</u> columns) of the number and the depression of one of them and the elevation of the other, and because of the increase in
11 the nearness to the earth or because of the increase in the numbers which estimate the magnitude, or something like that.
12 And this includes the sun and the centers of the epicycles of the planets.

68:13 However, as for the epicycle, its explanation should be freed first from the motion of its
14 center. And when we imagine it quiet and the planet on the perimeter moving,
15 the situation of the moon in it will be like the situation of the sun in the deferent.
16 And its motion in the higher segment will be seen (in a direction) opposite (to the signs) and in the lower segment
17 along the succession (of the signs.) But the situation of the planets in it will be contrary to it (i.e. the moon).
18 I mean, in the higher part along the succession and in the lower (one) contrary to the succession.
19 And if then the motion of the center is combined with it, and it is always along the succession,

69:1 conditions will vary according to (the relation) between the two motions, and the (higher) speed,
2 for the moon will be in the lower part, but for the planets in the upper part because of the addition of the two motions,
3 I mean the motion of the planet and the motion of the center (being) in one direction.
4 However, in the upper part, for the moon the two motions have different directions;
5 and what characterizes the motion of the moon due to the deferent goes analogously with the motion of the center.
6 And hence there results a decrease in the motion of the moon from the motion of the center, and that decrease
7 is a reason for the slowing down. And because of this the increments and the decreases in its sectors become like what
8 has preceded in (the case of) the sun, and I need not repeat it.

69:9 However, in the lower segment for the planets, their motion in it (is then) contrary to
10 the motion of the center. And it is known that the argument of the motion of the planet from the deferent, when
11 it is less than the motion of the center, it does not differ from the moon's necessitated motion in
12 the upper (part) of its epicycle being impelled to slow down. And when it is equal to it
13 it necessitates stopping, because of the equality of the two motions in two (opposite) directions. And when it is more (than the other)
14 and contrary to the succession, there can be nothing, after stopping, other than to retrograde. So the travel
15 of the planet will therefore be forward in the first and fourth sectors.
16 However, in the fourth it goes from slow to fast, (velocity), while in the first
17 from fast to slow. Moreover, in the second sector when it is before the first station
18 it is in forward (motion), and tending to slow down, and after it it retrogrades, tending to speed (up, backwards).
19 But in the third sector, (when it is) before the second station it will be retrograde

70:1 and tending to slow down in it (i.e. in retrogradation), and after it it (will be) in forward motion and tending to speed up in it. And the relation that
2 [God] be He praised!, has set between the motion of the sun and the motions of the planets
3 in the epicycle connects the matter of their retrogradations with the sun.
4 But the ancients did not portray this retrogradation with its true cause (as arising) in the eccentric orbit

70:5 or the epicycle. Perhaps they did not (want to) picture it for their public in a way that would be hard for them to understand.
6 And so they explained it to them as (being due to) halters joining them (the planets) to the sun. And this is why their followers have claimed
7 that the slackening of the planet's cord is in the two odd sectors,
8 and its [tightening] in the two even (ones). And they have assumed that when this halter tautens and tightens, it moves the planet
9 from its direction while retrograding, and when it tightens another time it drives it from retrogradation
10 to direct motion, and that is by attraction and slackening. And this (opinion), silly as it is, might be assumed in the case of Venus
11 and Mercury (to be) like a swing, pulled by a rope from the extremes of its swinging
12 on both sides.
13 But in the (case of) the superior (planets), I wish I knew how the halter could be equal to the amount of
14 the first and the second stations. And how does its tightening at them increase after being taut, where nothing
15 beyond this can occur except breaking and severance? And if the tightening has moved it from direct motion,
16 how can it increase after it; and why does the retrogradation not persist with the slackening of the cord after its tightening?
17 However, the situation of the equation with these sectors is as what preceded with the apogee (sectors).
18 I mean, it is increased in the first and the second and decreased in the remaining (ones).
19 But the computation is the reverse of what obtains in the (case of the) deferent, I mean it is increased

71:1 in the first and second sectors and decreased in the
remaining ones. In the matter of
2 light and size there follows for them what has per-
tained to the apogee sectors. They resemble in it the
latitude in
3 the quadrants of the inclined heaven, thus it,
(starting) from the ascending node, in the two odd
quadrants will be
4 increased, and deficient in the two even (ones). So it
will be ascending in the first and fourth quadrants
5 in both of its directions, and in the remaining ones
(i.e., sectors, it will be) descending in both of them.
And resembling it are the quadrants of the celestial
sphere as well as
6 the horizon. Thus the first quadrant is from the
ascendant in the direction of midheaven,
7 and the third quadrant, which is opposite it (is)
increased because of the coming of the day in one of
them and the coming of
8 the night in the other, and because of their approach
towards the meridian. And the half
9 which has the ascendant in its middle might have been
called increased totally because of its rising from
the nadir
10 to the zenith, and the other half (might have been
called) diminished. So these are the divisions
11 of increase and decrease according to those who use
them in both professions (astronomy and astrology?).

12 Mention of the Thickness Transit

13 Since the distances of a planet in its two
heavens differ, there being for it a greatest distance
14 and a nearest distance and a mean distance, which
(latter) is the mean of the (other) two, between those
(are) distances of various magnitudes

71:15 by combination (of the effects of the two heavens) and
individually. Any planet which is nearer to its
farthest distance in its sphere is defined as
16 transiting over that (planet) which is farther than
(the first) from its farthest distance in its sphere,
17 even though the order of the sphere of the one tran-
siting over (is) the inferior one. And when they
become equal in
18 nearness from the farthest distance neither of them
will transit over the other. And it was said that
they follow
19 a single course, regardless of the difference in the
order of their two spheres. So it is evident that
those who agree

72:1 upon this arrangement have not considered in it below
or above absolutely or additively, but
2 relatively (with respect) to the distances. Since if
they meant the absolute, the one having the inferior
sphere would never transit above the one of
3 the higher sphere. And if they meant the additive,
then let the centers of the two heavens of the planet
be imagined as
4 concurrent. There would then be for them in the
transit no additive above or below either, except after
5 equality of the two heavens. Because if they were
different, then let the planet of each one of them be
6 in its apogee or each one of them in its perigee, where-
upon there would be no alternative to the
7 transiting of the one with the wider orbit over the
one with the narrower orbit. But since the matter
8 is relative, they would revolve together in their paths,
meaning that each one of them
9 in its orbit is at the same distance if there is
assigned to the farthest distance a fixed number which
does not vary.
10 And if the case is so, the matter of the transit
becomes suspended (i.e. indeterminate); perhaps the
planet in

72:11 both of them (i.e. both the deferent and the epicycle)
is descending, those being dependent upon the unmodified center and the true anomaly.
12 Or perhaps the planet is ascending in both of them, or descending in both of them, or in one of them
13 ascending and in the other descending. Then the ascent and descent (may be) equal and in agreement,
14 which is rare; more commonly they are different and of two kinds, one (to be) added
15 to the mean distance so that the first and fourth sectors will be
16 ascending and the remaining ones descending. The other (type is) related to the apogee and its opposite (point)
17 so that the first and second (sectors) will be descending and the remaining ones ascending. And to this
18 the users of the transit, have referred, especially in their operation of projecting the (astrological) rays. So they consider
19 the mean (longitude) of the planets (along) with their true longitude, and when they find it less than the true longitude they claim

73:1 that it is descending, and when it is more than it they claim that it is ascending. And because of the equality in direction of
2 the epicycle centers of Venus, Mercury, and the mean position of the sun, this consideration
3 for the two (inferior planets) is by (comparing) the mean longitude of the sun and their true longitude(s).
4 Perhaps it is according to one of the zījes of the Hindus and the Persians in which
5 the mean (longitude) of each one of them is the sum of the mean of the sun and its (own) anomaly. And if the
6 difference between the mean (position) of the sun and its mean, meaning the mean of the planet, is taken, according

73:7 to what is in some of the books, there results the unmodified anomaly. And if we use (this) instead of
8 the mentioned difference, for the superior planets, between their mean and true longitudes, the result
9 deviates from its original (value) even if ascent and descent in the epicycle are determined through it. Then, even if
10 the excess by which the true longitude exceeds the mean or lags it were resulting from a
11 simple single equation, this consideration would have been sufficient, but it is (in fact) composed of two equations, one of them
12 from the deferent and the other from the epicycle. So there may result for it one amount.
13 And the planet for one of the two of them might be increasing by computation, and in the other deficient
14 in it, and then the two (might) go by cancelling each other so that the true longitude would neither exceed the mean nor be less than it.
15 But this would not be an indication that it is not ascending or descending.
16 Moreover, the true longitude might be in excess by two equations in those two directions,
17 differing in usage (i.e. sign) and unequal in magnitude, so that the excess would result from their difference,
18 or they might be in agreement in sign, and then its result would be from the sum of the two, or it might be from
19 an equation (in) one of the two directions only, without the other. But in the difference between the mean (longitude)

74:1 and the true there is no indication of a technique for that or of detailed knowledge about it.
2 And here Abū Maᶜshar's foot slipped after he had mentioned what we have just said.

74:3 That is that he added the maximum deferent equation to the maximum epicyclic equation
4 and made half the sum a basis for the consideration of one of the two of them in the determination of the planet's position.
5 He added them if their increases were together, or if they decreased together, and he took their difference if
6 one of them was increasing and the other decreasing. Then he compared the result with that base (value),
7 and if it were larger than it, he claimed the planet was ascending, and if it were less than it, the planet was descending, and
8 if it were equal to it, it was at its mean distance.
9 And let ABJ (Figure 13) be the heaven of the apogee, with center Z, and the center of the universe H,
10 and the midpoint between them Ṭ. And it is evident that B is (at) the mean distance
11 in the deferent. And let the epicycle [pass through] it when its center is at
12 one of the two points [R] (or) D. So it is apparent that if the planet were at B,
13 it would be in the mean distance in both heavens.
14 But, since Abū Maʿshar used the maximum equation of the center, we
15 dropped, to (fix) its position, HK normal to AJ, and let the epicycle center be at it.
16 And we extended HḤY tangent to it, and we made angle KHL equal
17 to angle [J]ZK. So arc YL of the pareclipic (is) equal to the sum
18 of the maximum equation of the center and the maximum epicyclic equation, approximately, and that is (because)
19 the position of the maximum equation of the epicycle, by verification, is point Ḥ. However

75:1 the equation of the center there is inaccurate, and hence it is improper to add both equations at one place,
2 they being at their maximum magnitudes. Let us bisect each one of the (two) angles YHK (and)
3 KHL, and then the sum of the halves will be angle ΓHʿ, and it is the base
4 which he put for consideration.
5 But this operation of his is irrational, in which he erred because this base was put for the mean distance,
6 and (he makes) the increase over it the ascent, and the decrease below it the descent. For, let the epicycle
7 be at Ṣ, and it is evident that the center at it, which is angle ZṢH, is capable of being
8 equal to half angle ZKH, or less than it, or more than it.
9 And if it equals it, and then there is added to it an equation from the epicycle in its lower part,
10 it (being) greater than half angle HK[Z], the sum will be in excess of the base.

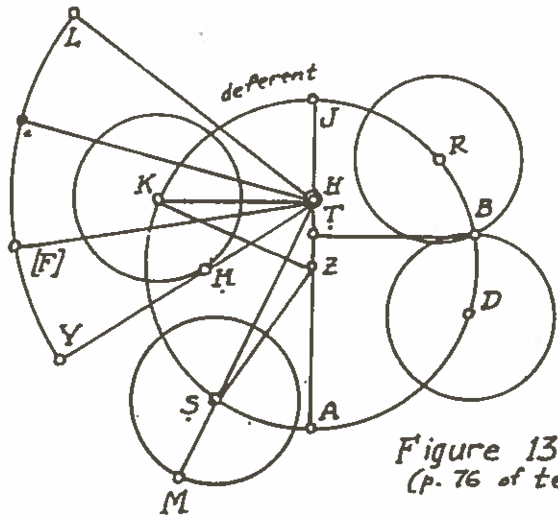


Figure 13
(p. 76 of text)

75:11 And it indicated the ascent of the planet which is descending in the epicycle even though, under these circumstances,
12 point Ṣ might be between the two points J (and) K, the
13 sum would be in excess (i.e., positive) and the planet in both orbits descending, because he put ascent
14 relative to the mean distance and descent relative to it. And if the planet is at point M,
15 with no epicyclic equation, and angle ZṢH, which in our case fails to
16 equal the base, then (the rule) indicated descent of the planet whereas indeed it had risen to
17 the epicyclic apogee and the center had not moved (at all) yet. However, the mark of the planet being at B
18 and its opposite, which two are at the mean distance, so if he had added the arc(sine) of half the eccentricity
19 to ninety, and took by (that) amount the equation of the center, and added it to twice

76:1 the arc (sine) of one fourth the diameter of the epicycle, which is approximately equal to the sum
2 of the maximum equations of the apogee and the epicycle, and made them the mean distance at D
3 and its opposite (point) on the other side in the third sector, the sum
4 of the two equations would be measured by them if both increased together or diminished together.
5 And he takes also the excess of twice the arc (sine) of one quadrant of the epicycle over the arc (sine)
6 of half the eccentricity, which is approximately equal to the difference between
7 the maximum of the equations in the two orbits; and we make it an indication for the mean distance at D and its opposite (point)

76:8 in the fourth sector in order to measure by it the difference between the two partial equations if one of them
9 is in excess and the other is deficient.

77:1 And he had then made very many artifices for it which were of no use
2 for it except considering the equation in each one of the two orbits with respect to the greatest one in it.
3 And it is computed at the position of the extreme of the equation (as) a mean distance. Because one who is better than him,
4 namely Abū Jaᶜfar al-Khāzin, omitted this or it just evaded his attention, and his evading of it is the
5 more probable for him in this situation because he mentioned it in the Ṣafā'iḥ Zīj. And he criticises
6 Abū Maᶜshar's saying that, "Some of the ancients dealing with the profession of astrology
7 "sought knowledge of the planets, but not many of them have determined its truth, but we have considered it
8 "until we have extracted it and explained it and put it in our zīj". And Abū Jaᶜfar
9 expresses his amazement at him because he did not add to what has already been done by those who preceded him except in explaining some of the numbers used in it,
10 as what we shall say in detail. Then he says that there resulted for Abū Maᶜshar the sun's chord
11 and according to its amount it ascends in the ether, because its ascent and descent from its mean distance
12 from the earth is according to the magnitude of the sine of the total equation, which is equal to the eccentricity.
13 So the ratio of the equation to its maximum becomes equal to the ratio of what belongs to the position of that equation
14 in the chord to its maximum.
15 And we will make it clear for him from what results for the partial equation, in whichever one of the

77:16 four sectors it may be, behold it is at its mean distance from the earth. For there is neither equation
17 nor chord for it at this distance. And this from Abū Jaʿfar is unsatisfactory. And he is to be criticized
18 in a manner by which Abū Maʿshar is not criticized, because of the difference in rank between them.
19 So we say to him, let the increase or decrease of the equation correct it in the half of the orbit

78:1 which is related to it, what then will correct it according to what is indicated by the sector and at
2 the mean distance, at which position, in fact, it attains neither the extreme equation nor the extreme chord?
3 And suppose further that the mean distance were at the position of the extreme equation, and so on both sides of it
4 in the two sectors (are) two positions at which the equation is the same and less than its extreme, and
5 in both of them it will be increased or decreased. So what distinguishes between them so that (one) can be led by it to distinguish
6 the sector? Then we ask him about what he said concerning the vanishing of the equation and the chord at this place,
7 which we have agreed with him to be the mean distance. For they reach at it their maximum values.
8 And what a difference there is between the vanishing of a thing and its attaining its maximum value (precisely), no
9 more and no less!
10 But Abū Jaʿfar still persists in the sharpness of his pen and his frequent carelessness which made
11 him slip many times and say what he did not verify. And Abū Jaʿfar knows, also,
12 that if he uses in (computing) the maximum equation certain numbers, some of which he multiplies

78:13 and divides by others, then if these same (numbers) are used in (computing) the partial equation, according to his rule,
14 a ratio will relate the two results. Such a thing astonishes us coming from Abū Jaʿfar, without his
15 accounting for the ratio. Such an operation makes the two of them alike in uselessness.
16 And we return after this to what we have been considering, and say that the belief of the (above-) mentioned
17 people concerning each one of the supposed rays in the aspects of the six planets
18 is a known quantity. If the planet is at its mean distance it is projected from
19 it at its mean position (or projection). Then it rises from it by its ascent and it depresses by its descent.

79:1 And an example of this talk of theirs is that a planet in the first of Cancer, for example, if
2 that were the position of its mean distance, and (if) its two quartiles fall on the two points of the equinoxes.
3 Then the first of Cancer, provided it is the position of its apogee, will it project the light of its two quartiles, the right(-hand side) being
4 into degrees of the first (part) of Aries, and the left(-hand side) into the latter part of Virgo.
5 But if the first of Cancer is at the opposite (point) of its apogee, the right one is projected to the latter part of Pisces, and the left into
6 degrees at the beginning of Libra. And they have announced that in saying that if the planet
7 goes down from the middle of its sector, that is, its mean distance, it sends its light and thus descends; but if
8 it rises from the middle of its sector, it sends its light to it and so it hastens. And it is evident from this that

79:9 ascent will be in the first and fourth sectors and descent in the remaining ones. But
10 they have operated contrary to this principle. That is that they considered descent as being increase of the true longitude
11 over the mean, and made it in the first and second sectors, and (they made) ascent to be decrease of the true longitude
12 below the mean, and that (is put) in the two remaining sectors. Then they made rules as to the magnitude of the transit.
13 The meaning of magnitude of the transit is that (distance) which each planet of the two rises or descends in its orbit.
14 Because if they become equal in ascent and descent, it is not said of either of them two that it is higher
15 than its partner or lower, that it (the magnitude of transit) be added to the true longitude if it is more than the mean,
16 and subtracted from it if less than it. Thereupon the operation of projection of the rays is performed upon the result.
17 And they may call it the body of the planet. And the true longitude which we have taken as an example in the first of Cancer,
18 if we require that it be greater than the mean, we need to add the magnitude of transit to it.
19 And if we do that the resulting left quartile will fall at the beginning of Libra because

80:1 of sending the ray. But the right one resulting falls at the beginning of Aries, extended. Because
2 if it were sent it would have fallen at the latter (part) of Pisces. And consideration of their first dictum without
3 their operation requires adding the transit to the true longitude for the left ray and its decrease

80:4 from the true longitude for the right ray. But consideration of their operation without their (stated) principle prescribes depression
5 of the left one and elevation of the right, although Māshāllāh operates with the left ray,
6 then he puts the right one opposite it and he does not operate with it.
7 But the investigation in the matter of the rays is separate from this art, although they have connected it to it.
8 As for the magnitude of transit, it is based on (the idea) that it is a part of six parts and one quarter of a part of
9 the difference between the mean longitude of the planet and its true longitude, I mean, four parts of
10 twenty-five of it. So when this difference is divided by six and one fourth by multiplying by
11 four and dividing the product by twenty-five, the result will be what is required.
12 And Abū Maᶜshar related this about those who had preceded him, (who also) doubled these two numbers
13 and performed multiplication by eight and division by fifty.
14 And what I find in the books differs in the expression of the numbers and in increasing them
15 and in doubling the ratio and complicating it. So what Māshāllāh explained, which is in agreement with what is in
16 the Shāh Zīj and al-Jawzaharī's zīj, is to multiply the difference by eight hundred
17 and divide the result by three thousand six hundred; and the quotient is multiplied by three hundred
18 sixty and the product is divided by five hundred, and the result is what is required. And it may be
19 that these people have a reason for increasing these numbers which we do not know, and until we determine (it) we

81:1 will take it as being a preference for abandoning that
which is limited (in the sense that) it has few
numbers in favor of that which has
2 many numbers. As for the (above-)mentioned ratio for
the determination of the required, it is composed of
3 the ratio of three thousand six hundred to eight
hundred, and of the ratio of five hundred to
4 three hundred and sixty. But the first ratio is the
ratio of nine to two,
5 and the other ratio is the ratio of twenty-five to
eighteen. And that is that if
6 we divide nine by two there results the ratio between
them reduced to one (in the denominator),
7 and that is four and a half, I mean four times and
half a time. And if we divide twenty-five
8 by eighteen the quotient will be one and seven parts
of eighteen,
9 out of one, I mean (one) time and a third and half of
a ninth. And if we multiply one of these two
10 resulting ratios by the other there results two
hundred and twenty-five to
11 thirty-six, and after cancelling between them it
becomes twenty-five to four,
12 and they are the two numbers of the base ratio.
13 And Abū Maᶜshar used for the planets the two
numbers of the first ratio
14 of the two constituent ratios, two and nine, and the
two numbers of the other ratio,
15 thirty-six and fifty, and that is twice what is neces-
sary for it.
16 But for the two luminaries he changed the first
ratio by making the number of the true longitude
17 for them four instead of two, and thus he ended with
twice what
18 the two produced. Perhaps he was led to that by an
idea which is unknown to us, and he imagines from it
that

81:19 he sought in his operation the equation due to (the
distance) between the center of the universe and the
center of the deferent. But

82:1 the ratio, according to him, for the orbits of the two
luminaries, they (the orbits) being the carriers (of
the two luminaries), have the ratio of two times and
one fourth a time,
2 I mean the ratio of nine to four. And he used it as
it is. Then, since the center
3 of the deferent in (the case of the) planets is mid-
way between the center of the universe and the center
of the equant,
4 he uses half of four so as to get from the equations
which are engendered at
5 the center of the equant, half what he would have
obtained had he used for it
6 the four as it was. But the result of that was made
to become what is imposed by the deferent center, not
the
7 equant. And what a resemblance can be drawn between
the person who moves from the simple ratio to a more
complicated one and the person who has
8 been invited to the happiness in paradise and refuses
to enter except after making the (required) pilgri-
mage (to Makka). But if it is a virtue to complicate
9 the operation by the insertion of an intermediary
between the difference and the required (thing), then
it is twice virtuous
10 to insert two intermediaries, thus having the ratio
composed of three ratios,
11 and it would come in eight numbers instead of six.
God forbid,
12 since increase in the work is a decrease in precision
and increasing this is the putting of
13 one burden atop another.

82:14 And in some of the books of the astrologers a
method is found according to which the difference is
multiplied by
15 forty and the number is then divided by a hundred and
eighty, and the quotient is then multiplied by
16 eighteen, and the product is divided by twenty-five.
However, the two numbers of the second ratio
17 have the same form (as the previous ratio), whereas
the two numbers of the first ratio (are) each equal
to twenty times
18 what is required, and the result is correct and
unchanged. And in some of them
19 the two numbers of the first ratio are found also,
multiplied by twenty. But (in the case of) the numbers
of the second ratio

83:1 the first is three parts and three fifths, I mean, two
hundred and sixteen
2 minutes, but the other (is) five minutes. And this
seems to be a slip of the copyist,
3 because when he saw the first number in minutes he
thought that the second is thus also, and so
4 he assigned the same (unit) to it. But the five are
parts, in fact, and not minutes.
5 And no regard should be paid to the variants of
the copies and the errors of the copyists, for Abū ʿAlī
al-Shāhid mentioned
6 this same thing, but dropped eighty from the divisor
in the first ratio,
7 and it became one hundred in his edition. And also in
some books of Māshāllāh
8 the thing multiplied in the first ratio has been
changed, and it was made (into) one hundred and sixty,
and that is four times
9 the forty, but the divisor in it was left as it is,
that is one hundred and eighty. And they were both
corrupted
10 in copying some of his books. So he made the first
sixty and the second eighty-eight. And they are

83:11 in the ratio of fifteen to twenty-two. And if the
first were made thirty times
12 as much as is necessary, it would then be necessary to
do the same for the second so that it becomes
13 two hundred and seventy. And the result of all these
corruptions is bad and they
14 are different in the numbers.
15 And al-Farghānī has mentioned, in this connection,
to transform the whole [excess] into minutes and to
multiply
16 by forty-eight minutes and to divide the product by
five, and the result would be in seconds. And then
17 he doubles what remains and multiplies by six, and it
becomes thirds. This agrees with what preceded
18 concerning the ratio between four and twenty-five.
For when he took a fifth of

84:1 the twenty-five, he took also one fifth of four, and
that is (the thing) by which (he) multiplied (it).
2 And because division is by five and the remain-
ders are parts of it, but sixty is what is intended
3 without five. And if twice five is multiplied by six
(it) will be sixty,
4 and it is the divisor. And we should treat the remain-
ing (ones) thus so as to have the ratio come back.
And if he had
5 taken one fifth of one fifth of twenty-five, which is
one, and separated from four one fifth of its fifth,
6 and that is nine minutes and three fifths of a minute,
and then multiplied what remains by
7 five hundred and seventy-six seconds, he would have
reached the first. And he would not dispense with
division
8 by elevating the result sexagesimally.
9 Also multiplication (is) by forty-eight minutes
and division (is) by
10 five; but what was multiplied by twelve minutes had
been divided by five, and it looks as if

84:11 the difference is in need of two multiplications: one by forty-eight minutes, and the other

12 by twelve minutes; and the product of one of them by the other (is) nine minutes

13 and three fifths of a minute. And if the difference is multiplied by it there results what is required.

14 And then (the above) was found in the talk of Māshāllāh about the <u>Book of Conjunctions</u> by

15 Ibn al-Bāzyār.

16 And in Ḥabash al-Ḥāsib's zīj there is a sugges- tion regarding this (and that is) of multiplying the excess (difference)

17 by seven instead of the four there, and dividing the number by twenty-two instead

18 of twenty-five there, and there results what is required. And he had suggested in some of the copies

19 doubling what comes out, whereas halving it is more relevant, since the result would be close to

85:1 twice what results from the base ratio.

2 And I do not know from where they have taken this ratio. It seems as if they had sought by it

3 to curve a straight line and bend a plane. But what is more strange is what I have read in some of the manuscripts

4 of the Shāh Zīj of using the ratio between four and twenty-five

5 for the superior planets and using that between seven and twenty-two

6 for the inferior ones, thus introducing innovations "queerer than the croaking crow".

7 And Ibn Muḥammad in his al-Kāfī Zīj, has dropped ten out of the

8 twenty-two and made the division by twelve. And some of those who perform this operation

9 have composed a table for the transit from one to sixty and computed it according to the preceding calculations.

85:10 But there is nothing in the equations of the apogee more than what there is for Mars in the Canon, and there is nothing in

11 the equations of the epicycle more than what there is for Venus in the Shāh Zīj, and their sum, even though they are not added, is less

12 than sixty. So there is no restriction, so far as these number(s) are concerned, to the assumed maximum (size) of the transit.

13 (And there is nothing to explain) with regard to that except to say that what he put in the table are the arguments of the degrees,

14 adjoining them (the degrees) in the column of the argument. And if he regarded these degrees as minutes, (the entries) which correspond to them

15 in the table are rearranged by putting a zero above them, they are for their arguments; and if it is

16 seconds, what is opposite it is rearranged by putting two zeroes above it, would also (be)

17 its argument. And the table includes what he needs for the morning (sic) and its accessories.

18 However, this magnitude which was obtained for the transit from the difference (which is) combined of

86:1 the two equations, was forsaken by Abū Maᶜshar, who took instead the components and performed for each planet

2 at its maximum equation the operation we have mention- ed. And he called the result the chord of

3 that planet, related to the apogee if it had been performed with the equation of the center, [and] (related) to the radius

4 of the planet if performed with the equation of the epicycle. And he put them (as) bases. Then he opera- ted

5 with the equation of the center and the anomaly each, (which are) the two parts in the determination of the true longitude

86:6 of the planet, like that operation, so that he obtained the partial chord for it; and he
7 divided it by the chord and called the result minutes of transit of the planet from the chord. And it,
8 in the first sector, is that which, being divided by the equations, is the magnitude of its descent from its apogee to its
9 transit from the chord. So if the minutes of the apogee equal the minutes of the chord, its transit would be
10 at the beginning of the second sector. And he subtracted in this sector the minutes of transit from the chord, so there remain
11 the magnitude of its descent in the chord. So if the minutes of transit are null, its transit will be at the beginning of
12 the third sector. And the minutes of transit in this sector measure the magnitude of its ascent and transit
13 in the chord. And in the fourth (he) subtracted the minutes of transit [from] (those of) the chord, and there remained its ascent in this
14 sector and its transit in the chord. And it is apparent that he takes of the maximum equation four
15 parts of twenty-five of it, and he measures by it its ratio of the partial equation.
16 And the ratio of the part to the part that is named after it is as the ratio of the whole to the whole. So either
17 he did that or he measured the partial equation as it is to the whole (one) as it is; and what
18 he got from these chords (is) what we have put in this table: (Figure 14, on the next page.)

87:1 And this is Abū Maᶜshar's method regarding the transit of the planet from the chord. However, as for
2 the transit of the planets, one across another, according to his description, it will be for two planets

☉	☽	♄		♃	
Apogee [Chord]	Radius [Chord]	Apogee [Chord]	Radius [Chord]	Apogee [Chord]	Radius [Chord]
[0];42,42,36	1;34,43,12	1;22,43,42	[0];44,42,48	[0];48,44,36	1;44,19,42
♂		♀		☿	
Apogee [Chord]	Radius [Chord]	Apogee [Chord]	Radius [Chord]	Apogee [Chord]	Radius [Chord]
1;47,12,[0]	6;23,48,[0]	[0];42,12,38	6;32,12,36	[0];38,24,[0]	3;26,24,[0]

Figure [14] (p. 91 of text)

87:3 that are together in the two superior sectors, or together in the two inferior sectors. But he does not consider it
4 when one of the two of them is in a superior sector and the other in an inferior sector, or for those that differ

88:1 in two sectors, even though they are in one direction.
2 And these are the meanings of his saying in his zīj: "Verily transit between two planets which are in aspect
3 "is divided into two parts. One of them is that they be together in (one of) the two superior sectors,
4 "and the second that they be together in (one of) the two inferior sectors. And that is either in the deferent
5 "or in the epicycle. So that has four cases.
6 "And their order in strength is that priority (is) to the superior deferent (sectors), then to the superior
7 "epicycle (sectors), and then to the inferior deferent (sectors), and then to the inferior epicycle sectors. And its order

88:8 "in strength of aspect (is): conjunction (has) priority, then opposition, then the quartile, then the trine, and then

9 "the sextile, except that the last two are weak, and the upper one of these two planets,

10 "which transits over the other is the one nearer to the epicyclic apogee. If they become equal, neither

11 "transits over the other. And if the lower one of them becomes lowered by the amount of the minutes of its body (i.e., apparent diameter) which are perceived

12 "by vision as put for it in the table of its equation in that zīj. Then, if it passes it,

13 "the transit becomes weak and it goes on getting weaker and weaker until their two sectors become different, and then it disappears."

14 And this is the gist of his talk.

15 And in this he is an innovator and a reconciler by his innovations, since how could the transit vanish

16 due to the difference of the sectors, since the upper two are precisely those above the lower ones,

17 and the [distances] of the one sector are different; and whichever of them is nearer to the epicyclic apogee

18 (is) above the one below. But the transit vanishes at the equality (of the equations?) due to the vanishing of the elevation (of one over the other).

19 And it is non-zero when there is a difference and increases in magnitude with increase of the difference. So, if the transit

89:1 is nonexistent, the (distance) between the two transits will increase, and so it is more logical for it to vanish due to the increase in (the distance) between

2 the two (heavenly) bodies, and opposition at it (transit) is stronger than the quartile and it is farther from it in magnitude

3 and distance.

4 And what he ought to have done in the requirement of the magnitude of the planet is to make it

89:5 (equal to) half the sum of the two magnitudes, and it is at the time of tangency, if we imagine them (to be) in one heaven

6 he (must) remove from it the matter of parallax, since he needs it in what relates to it

7 of uncovering and eclipsing. But there is no use in following the discussion about

8 that, and we shall do it when considering his zīj, if God delays our due time and helps us do it!

9 And there is no doubt but that ʿUmar ibn al-Farrukhān and Māshāllāh are midway

10 between Abū Maʿshar and the Persians his leaders. And their words, which are confused and self-contradictory,

11 are not worth mentioning, (yet) it is well to state them for two reasons:

12 One of them is to make it known that Abū Maʿshar does not agree with them, and the second is to drive the reader away from him,

13 lest he should think well of him, and imagine from its non-appearance in our talk, that we did not find it.

14 And so let us say what we found concerning that.

15 ʿUmar said: "Transit exists only in conjunction and opposition and the quartile and then it is weakened for the

16 "trine and sextile. And the excess of the true longitude over the mean is the indication of descent

17 "of the planet, and being less than the mean is an indication of its ascent. Then present the equation(s)

18 "of the two planets to determine which planet is the higher in transit, and find it for each

90:1 "one from the second, deferent (? jawwī) and chord, meaning the eccentric and the epicycle, and use

2 "each one of them with (its) opposite (i.e., its corresponding one of the other planet). And if the planets ascend together or descend together in one

90:3 "of the two categories, take the difference between their two equations for it. But if one of them ascends while

4 "the other descends, add their two equations for it, and divide the result of that by the apportionment

5 "between the two planets. Thus there will come out the magnitude of the elevation. And the consideration for it, for each sign (is)

6 "six. As for the apportionment between the two planets, it is found by taking the eccentricity

7 "of each one of them and dividing the larger of them by the smaller, and what results

8 "is their apportionment."

9 But verily Māshāllāh divided the maximum equation of the epicycle and its numbers for each,

10 and we have put both types, in two [pulpit](-shaped arrangements), according to the Shāh Zīj, for the equations

[Apportionment] between the Planets in the Deferent

[Apportionment] between the Planets in the Epicycle		♃/♄ 6823	♂/♄ 2544[0]	♀/♄ 2962[5]	☿/♄ 13500
♄/♃ 6082			♂/♃ 13422	♀/♃ 1[5]6[31]	☿/♃ 7123
♂/♄ 4679	♂/♃ 7896			♀/♂ 4[1]92	♂/☿ 6784
♄/☉ 13994	♃/☉ 8283	♂/☉ 18217			♀/☿ 7900
♄/♀ 13994	♃/♀ 8283	♂/♀ 18217	☉/♀ 1[P]		
♄/☿ 7755	♃/☿ 4590	♂/☿ 10010	☿/☉ 6496	☿/♀ 6496	
♄/☾ 6288	♃/☾ 3722	♂/☾ 8180	☾/☉ 8012	☾/♀ 8012	☾/☿ 4440

THE [CHORD]

THE DEFERENT

Fig. [15]

(page 87 of text)

90:11 after we changed into seconds for simplification.

12 And it is apparent from ʿUmar's words that he divides the orbit, for ascent and descent,

13 by the diameter passing through the farthest distance and the nearest distance, and it is one of the (above-) mentioned opinions

14 regarding ascent and descent. But inferring it from the situation of

15 the mean and true longitude gives a different and invalid (result). For the difference between them may be in only

16 one of the two heavens, or it may be compounded of the sum, in agreement in both of them, or the difference of two differing (categories).

17 Hence, it is necessary to define ascent and descent for the deferent by what is between

18 the mean and the adjusted center; and for the epicycle, from what is between the center and the

91:1 true longitude. But the deferent differs in this sense from the epicycle, if

2 the motion is from its epicyclic apogee along (the direction of) the succession (of the signs).

3 And it is known from his operation that he uses the equations themselves without transforming them

4 by an operation which was previously (explained) regarding the dependence of the composite ratio in it, and Māshāllāh is in agreement

5 with him in that, and even more confused (than he was).

6 And before discussing his opinion, we state what helps in considering the problem from his (?) point of view.

7 Let the orbit ABJD (Figure 16) be divided by the sectors, AB, BJ, JD (and) DA

8 into quadrants, approximately, since this is not the place for precision, and the equation of the center

92:1 according to the Hindus and the Persians (is) divided at the quadrants of the orbit. And we extend AH[J].

92:2 its first diameter. So the apogee will be A or (it will be) the epicyclic apogee; and BHD is its second diameter.

3 And let the succession (of the signs be) from A towards B. And if we regard it as

4 the epicycle of the planet the true longitude will be, in the semi(circle) ABJ, in excess of the center (i.e. the mean longitude or anomaly),

5 while in the semi(circle) JDA less than it. But if we regard it as the deferent (of the planet) the

6 center will be less than the mean in the semi(circle) ABJ and more than it in

7 the semi(circle) JDA. And the equation in both orbits is

8 increasing in amount in the quadrants AB (and) JD, and diminishing in the quadrants BJ (and) DA, and that is, among

9 the people, computed by the (method of) sines.

10 And we have already mentioned that there are two opinions regarding ascent and descent. One of them

11 considers ascent (to be) in the semi(circle) DAB, but the other considers it (to be) in the semicircle JDA.

12 So let Z, a point in the first quadrant, be the position of a planet from which we measure

13 the positions of the planets. And we let planet Y be with it in (the same) quadrant, and we drop from them

14 the two perpendiculars ZḤ (and) YK, and their two transits (in thickness) will be at the two points Ḥ (and) K,

15 approximately, because the accurate determination would be to draw, with the center of the universe as a center and the distance of each

16 from Z (and) Y (respectively) as a radius, a circle (madār) such that their ends will be at the diameter AH[J] and their two transits will be on it.

17 Whereas according to the first opinion, their ascents will be HḤ (and) HK. And ḤK,

92:18 the elevation, is the difference (of the distances) which are between them. But according to the second opinion they are descending,

19 and their descents (are) AḤ (and) AK. And ḤK, the elevation is the difference (of the distances) which are between them.

93:1 So if the maximum equations are equal (?) for both planets, and in addition let its (what's?) position be at

2 epicyclic apogee, so that the argument of planet Z will be more and the argument of planet Y

3 less, it would be in agreement with the law of elevation, since it is the one having the greater equation. But

4 the actual situation is contrary to this.

5 And because it is possible that the total (i.e. maximum) equation for planet Z is

6 greater than the total equation for planet Y, the partial equation(s)

7 may be equal in amount at the two positions Z and Y, and even that

8 at position Y, might exceed that at position Z, even though the two total (equations) were equal,

9 as well as where that for planet Y is greater. But this is contrary to the law

10 for the elevated (ones). And the ratio of the difference between the two equations, at it, is to the amount of elevation

11 as the ratio of the greater of the two total equations for the two planets is to the smaller. Because

12 in apportioning, when the greater of the two total (equations) is divided by the smaller there results the ratio

13 to unity of that ratio. And due to this we do not multiply the difference between the two partial equations

93:14 by the fourth of the magnitudes, because it is one. But he (Māshāllāh or ʿUmar) divides it as it is by the next (number in the proportion), which is
15 the apportioning (coefficient). And the method is satisfactory if the excess is to the equation of the planet
16 having the greater total equation, but if the excess is for the equation of the other one
17 it is not satisfactory.
18 And we are now investigating (that) opinion which coincides with elevation, which is one
19 of the two opinions regarding ascent. So let us suppose planet L (to be) in the fourth quadrant

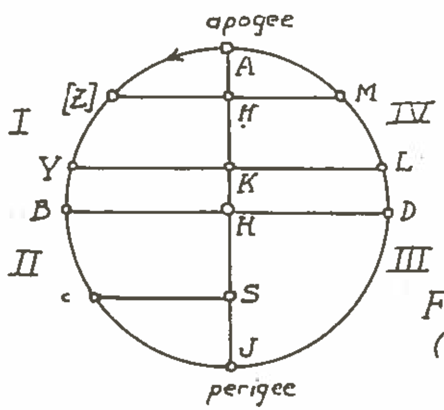


Figure 16
(p. 95 of text)

94:1 and its transit will be at K and its ascent HK, and the elevation of planet Z over it
2 (is) HK, I mean the difference between the two ascents. However, according to the latter opinion
3 it will not hold, because the descent of Z (is) AḤ, and the ascent of L (is) [J]K.
4 But ḤK is not the difference between them unless he calls AL, which is the supplement of [J]L, the
5 descent for it, so that the magnitude of descent will be (equal to) AK, and we make the required condition as

94:6 ascent for (that?) one of the differences (determined) with the agreement of the property (i.e. having the same sign) at its leg. Then we suppose the planet
7 in this quadrant (to be) at M, and then its transit with Z will vanish because of the equality of their equations.
8 And the elevation of planet M over L will be the magnitude of the excess ḤK,
9 and that is because of the agreement of the two planets M (and) L in the property of ascent. And thus is the case
10 for any two planets found together in one quadrant, according to the rule of the operation. Then we suppose a planet
11 (to be) at ʿ in the second quadrant. So its transit will be at S, and the elevation
12 of planet Z above it (will be) ḤS, which results from adding the ascent HḤ to
13 the descent ḤS, and verily it is in accord with the first opinion.
14 However, according to the other opinion, in which case they agree in descent, with the condition
15 for its validity being the taking of the difference, it is possible that ḤS be the difference between the descents
16 AḤ (and) AS. But if the descent AḤ occurs at the equation of Z,
17 then at the equation of ʿ, the only thing that can result is ḤS, and ḤS
18 does not result from the difference between AḤ (and) ḤS unless the sum of the two equations of
19 Z (and) ʿ is subtracted from the sum of their two total equations, and then we divide the remainder by the apportionment.

95:1 And according to this opinion, if the excess is for the equation of ʿ over

95:2 the equation of Z, (it) would be the elevated (one),
and it is the lower one, and (it is) the measurement
of what is in the quadrant DA.
3 But the case in an opposition, and (what) is between
the planets M (and) ʿ of elevation
4 is ḤS, which, according to the first opinion, is the
sum of the ascent of M and the descent of ʿ.
5 And the latter opinion requires addition also. What
on earth justifies
6 their addition? For the ascent of planet M is JḤ, and
the descent of
7 planet ʿ is AS. And it is necessary here to go back
to the first opinion and
8 to take ascent from diameter BHD (as) towards A, and
descent
9 from it (as) toward [J].

96:1 And after stating this we go back to the confu-
sions found in the books of
2 Māshāllāh, and we mention them with their difference(s,
i.e. their variants). Though it is more probable that
their cause
3 is the faults of the copyists and the ignorance of the
users.
4 And he said in his fifteenth book(?), "On the
Transfer of the Cosmic Years"(<u>Fī</u> <u>taḥwīl</u> <u>sinī</u> <u>al-ʿālam</u>),
5 like what was said by Ibn al-Farrukhān. And he took
also, in an example for Jupiter, one part out of six
parts
6 and one quarter of a part of the difference of what is
between its mean and its true position. And he added
it to its true longitude
7 if it was descending. And he derived the required
magnitude of the projection of the rays by the opera-
tion ascribed

96:8 to him, except (for) the opposition, which he took at
the true opposition, unmodified.
9 And it is apparent that this is in accordance with
the second opinion. But when it passed this position
10 he claimed that the planet (is) ascending in the first
and fourth sectors, and descending in the rest.
11 And this is in accordance with the first opinion. (Even
this would not have been so bad) had it not been
followed by a confusion, which is
12 his saying that that is for the three superior (pla-
nets), whereas the usage with the inferior (planets)
is to consider
13 their epicyclic sectors. And these are words void of
meaning. Since the five planets have in common
14 what demands for one of (any) two of them a deferent
and for the other an epicycle. And the two luminaries
share with them
15 one of the two of them. And no matter how ascent and
descent are taken, they are all
16 the same, and not differing except by the magnitude of
the sector because of the variation of the magnitudes
of the
17 total equations.
18 And if it is said about the true longitudes of the
sun and the moon, and about the adjusted center of the
planet,
19 that if it becomes less than the mean, then it is in
either the first or the second of

97:1 the deferent sectors, and if it is larger than it, it
would be in one of the remaining ones. But if the true
longitude of (one of) the planets
2 is larger than the modified center, the planet will be
in either
3 the first or the second of the epicyclic sectors, but
if it is less it will be in one of the others. Ascent

97:4 and descent were defined by those who take them from
the first diameter, I mean the farthest distance
5 and the nearest distance.
6 However, according to those who take them from
the second diameter, I mean the two mean
7 distances, they would not be determined except by compa-
rison between the unequated (or unmodified) center or
the equated (or modified) argument
8 and the magnitudes of the sectors as set in their two
orbits.
9 Thus this statement of Māshāllāh cannot be
interpreted except as meaning that
10 ascent and descent are in the deferent for the three
superior (planets), but for the two inferior (planets
they are)
11 in the epicycle.
12 And what follows in his book is still more
confused. For verily he said: "As to the transit of
the inferior (planets),
13 "they are up to six signs attracted from the orbit (or
circle, manṭaqa) downward, and in what remains they
are ascending."
14 And this is the second opinion, common to the majority.
15 Then he explained in detail what he had said in a
concise way, saying: "As for Venus, up to four signs
16 "and a half it is falling from the downward, and up to
six signs ascending from
17 "its fall from the manṭaqa, and up to seven signs and a
half ascending above the manṭaqa,
18 "and up to twelve signs descending from ascent (down)
to the manṭaqa." And he mentioned (what is) like it
for
19 the sectors of Mercury with their magnitudes. And it
could hardly be imagined from the ascent in the second
sector

98:1 and descent in the fourth except to replace by it the
maximum equation, increasing

98:2 to an extreme (value) for descent and decreasing to an
extreme for ascent so that ascent will occur in the
second
3 and descent in the fourth, according to, he says,
decrease from the extreme,
4 or recession from this (decrease), and this, praise
God!, (makes) a third opinion.
5 And he said, concerning the two luminaries, that
up to six signs they are ascending above the manṭaqa,
6 and in all that remain (they are) descending.
7 Then he explained the whole matter in detail also,
that up to three signs they are ascending, and up to
8 six descending from ascent to the manṭaqa. And up to
nine (they are) descending from
9 the manṭaqa downward. And in what remains (they are)
ascending from their descent.
10 However, the usage of quadrants is due to what we
have previously mentioned about the Hindus and the
Persians on
11 cutting (i.e. determining) the kardajāt of their two
equations and the equation of the center at the
complete quadrant.
12 Generally speaking, the first opinion (is main-
tained). But in the concise part (of his statement he
agrees) with the first opinion, because decrease
13 in the equation, if it indicates ascent in the epi-
cycle (it) occurs in the third
14 and fourth epicyclic sectors, whereas with the defe-
rent (it) occurs in the first and second sectors.
15 Whereas in the details he assumes the second
opinion. And how strange is this of him,
16 since his doctrine differs as between the summary and
the details. Thereupon, in what comes after that,
17 (he) said: "If the conjunction exceeds one minute,
"the transit becomes weak, but (it is) strongest when
it is in conjunction.

98:18 "However, in departing it is weak." And this, from him, is an indication that he considers a transit in
19 opposition as being in conjunction, with the sectors being different. "Then if it is receding, it

99:1 "becomes weak because of recession (insirāf), not because of the vanishing of the transit, since its vanishing (implies) the vanishing
2 "of the elevation, which occurs only at equality."
3 The place where Abū Maᶜshar fixed the transit, which does not occur except at
4 the place where he made it weak or null, is contrary to what (most of) the people do about it.
5 Māshāllāh was kind enough to produce an example of the year-transfer in which the transit passed from the earth triplicity
6 to the air (one), and its horoscope was (at) two thirds of the sign of Leo, and Jupiter was in
7 Virgo in twenty-two degrees and forty-four minutes. Saturn (was)
8 in Libra in nine degrees and eight minutes, and Mars (was) in Pisces in fourteen degrees.
9 And there is no use in mentioning the positions of the inferior (planets), since he did not use them,
10 as if their strength in important matters is little. And because the conjunction is in reception and Mars
11 (is) going to be conjunct with Mercury and the sun with Saturn (he) made some as elevated
12 over others, his opinion regarding it (being) different from that of Abū Maᶜshar, (which is) to fix
13 the transit between the two (planets) in conjunction, even though what is between them became farther in degrees. And he changed their two places
14 in the two sectors, explaining that the weakening of the transit will be by recession, and its vanishing

99:15 by not being in aspect or relation. He started with what is between Saturn and Jupiter. So he decreased the jawwī of
16 Saturn, that is its [apogee] from its true longitude, and he claimed that the tabular entry opposite the remainder is four hundred minutes
17 descending in the sector.
18 However, the apogee of Saturn in the Shāh Zīj (is) two hundred and forty parts. So the remainder would be
19 three hundred and nine degrees, and opposite both in the table of the equation of the center

100:1 for Saturn (the entry is) six parts and thirty-five minutes. And that is near to what was mentioned. Because
2 this remainder is not from the unmodified center, (i.e. it is from the adjusted center) so that this item will also be
3 its equation in reality.
4 It may be that his operation in getting the (above-)mentioned minutes was that he
5 took with the distance of Saturn from the sun, which is one hundred and seventy degrees and
6 fifty-two minutes, the equation of the argument for Saturn, and so it was one degree and eight minutes, and he subtracted it
7 from the position of Saturn. So (it, the position) became one hundred and eighty-eight degrees, and it is approximately the
8 modified center. Then he subtracted from it the apogee and took the equation of the center of Saturn with what remainde.
9 and it was six degrees and forty one minutes, as they (the users of the Shāh Zīj?) mentioned. And he subtracted it

100:10 from the center and added the apogee to the remainder and got one hundred and eighty-one

11 degrees and nineteen minutes, which is the approximate mean position. And the excess of the true longitude over it implies

12 descent according to what preceded in his operation. But a part out of six parts and one fourth

13 of it will be seventy-five minutes, and so it did not go into descent

14 in this manner. But rather the deferent was decreased, and Saturn, in it (the deferent, is) in the fourth sector.

15 And the adjusted center is more than the unadjusted, and it is therefore descending in it. Then he put

16 Jupiter in the position of Saturn, because it is going to be conjunct. But when they become conjunct he takes

17 their (common) position. And because of this it would have been better to perform his operation upon both of them at the part (i.e. longitude) of the conjunction.

18 And then he diminished the apogee of Jupiter, which in their zīj (the Shāh) is one hundred and sixty parts, from the longitude of conjunction,

19 and claimed that it is replaced by one hundred and forty-eight minutes ascending from the sector.

101:1 And that is equal to the tabular entry opposite the remainder, which is twenty-nine degrees

2 and eight minutes, and it (the entry) is two parts and twenty-eight minutes. And on separating out the two equations,

3 the equation of the center will be approximately two parts and nineteen minutes. Its ascent

4 (is) in accordance with the second opinion also because it is in the first sector.

5 And since Saturn and Jupiter differ in character (i.e. sign), he added what minutes they had

101:6 and divided the five hundred and forty-eight minutes by the apportionment between them,

7 according to him. So there came out (for) the elevation of Jupiter over Saturn five parts and twenty-eight

8 minutes. And he attempted after that Jupiter and Mars. As for Jupiter, he diminished

9 its apogee from its position and claimed that it gets sixty-seven minutes above the sector.

10 And that is close, because the tabular entry opposite the remainder, which is twelve degrees,

11 and forty-five minutes, is one part and nine minutes. And by taking the difference between the two equations

12 the equation of the center will be one part and eleven minutes.

13 As for Mars, he put it in the place of Jupiter and subtracted from it its apogee,

14 which is one hundred and fifteen parts. And (he) claimed that what it is is five hundred

15 and sixty-six minutes below the sector. And that is not far (off) as the tabular entry opposite

16 the remainder, which is fifty-seven degrees and forty-five minutes, in the table

17 of the equation of the center for Mars, (is) nine parts and twenty-seven minutes. But the difference between

18 the two equations is far from it.

19 And because of their difference in character (i.e. sign), he added what is for it and divided the six hundred

102:1 and thirty minutes by the apportionment between them, and the elevation of Jupiter above

2 Mars came out (as) four parts and forty-eight minutes. As for the descent of Mars, it is from the

3 side of its position from its apogee, regardless of the position of Jupiter in which he placed it. So

102:4 from there it is ascending, and by elevation by
[subtraction], not by addition.
5 And because of this, one might think from the
word of Māshāllāh that I put
6 Mars in the opposite (point) to that place that he
means (for) the position of Jupiter, and that he put
7 Mars in fourteen degrees of Pisces. But had he done
(that) the equation would have come out for him (as)
8 three parts and one fourth. And if he had it in
opposition to Jupiter as twenty-two degrees
9 and forty-four minutes of Pisces, the equation would
have come out for him as seven
10 parts and a half. And had he put it in opposition to
itself, in fourteen degrees of
11 Virgo, the equation would have come out for him as
seven parts and one minute.
12 But all these derivations are far from the
(above-)mentioned minutes.
13 And after that he worked on the sun and Saturn. He
had already finished with Saturn in its position,
14 so he placed the sun in its (Saturn's) place and
dropped from it its apogee, which is [eighty] parts.
15 And he explained that it is one hundred and twenty-
five minutes below the sector; it
16 is in agreement with the tabular entry opposite the
remainder, which is one hundred and nine degrees and
eight minutes,
17 in the table of its equation, and it is two parts and
six minutes, and he did not use it as being in the
first (point)
18 of Aries, I mean its (proper) place, nor in the first
(point) of Libra, I mean its opposite (point) in the
sign of Saturn,
19 because its equation at both of them (is) two parts
and ten minutes. And what I had predicted, as to its
direction,

103:1 for Mars was true, since by measuring its position
with respect to its apogee it is descending, I mean in
the
2 fourth sector with its mean less than the true longi-
tude. And by measuring the position of Saturn rela-
tive to its apogee, it is
3 in the second sector, ascending. And due to their
agreement in character (i.e. in sign) he took the
difference between them.
4 And it (the excess) is for Saturn. And he divided the
two hundred and seventy-five minutes by
5 the apportionment between them. So there came out one
part and ten minutes, and it is the elevation of Saturn
above
6 the sun. And the sum of what he got from the eleva-
tions, (is) eleven parts and twenty-six minutes.
7 Its duration, by measuring the <u>tasyīr</u>, is eleven years
and five months
8 and six days. He worked backward on it the two hundred
and forty years
9 which are for the transfer of the transit. And he
arranged them according to the strength(?) and the
witnessings
10 desired from astrology. And what resulted from the
example of Māshāllāh is that he sees
11 the transit as fixed in between the two conjunct
planets, and he sees its weakness by recession,
12 and its vanishing by falling, even though their
magnitude might be small. And he follows as to ascent
13 and descent the second opinion and not the first. And
thus I have made known the aims of the people
14 in their operations.
15 So let us put now in a table what has been
mentioned of the requirements of the sectors,

103:16 and the increments, and the decrements in them, to simplify working with them.

<table>
<tr><th colspan="2">THE SECTORS</th><th>1st</th><th>2d</th><th>3d</th><th>4th</th></tr>
<tr><td rowspan="5">What is common to the (heavens of) the apogee and the epicycle</td><td>Descriptive of the sectors themselves</td><td>descending محرر</td><td>descending هابط</td><td>ascending صاعد</td><td>ascending صاعد</td></tr>
<tr><td>Descriptive of the planets in them according to the first opinion</td><td>ascending صاعد</td><td colspan="2">descending هابط</td><td>ascending صاعد</td></tr>
<tr><td>and according to the second opinion</td><td colspan="2">descending هابط</td><td colspan="2">ascending صاعد</td></tr>
<tr><td>Light, magnitude, and number</td><td colspan="2">increasing زايد</td><td colspan="2">decreasing ناقص</td></tr>
<tr><td>The equation</td><td>increasing زايد</td><td>decreasing ناقص</td><td>increasing زايد</td><td>decreasing ناقص</td></tr>
<tr><td rowspan="2">Special to the apogee (heaven)</td><td>The computation</td><td colspan="2">decreasing ناقص</td><td colspan="2">increasing زايد</td></tr>
<tr><td>The motion</td><td>slow بطي</td><td colspan="2">fast سريع</td><td>slow بطي</td></tr>
<tr><td rowspan="2">Special to the epicycle</td><td>The computation</td><td colspan="2">increasing زايد</td><td colspan="2">decreasing ناقص</td></tr>
<tr><td>Motion of the planet</td><td>(from) fast to slow</td><td>slow, [then] stationary, [then] retrograde accelerating</td><td>retrograde decelerating [then] stationary, [then] fast</td><td>(from) slow to fast</td></tr>
<tr><td rowspan="2">Latitude in the two directions</td><td>The direction</td><td colspan="2">north</td><td colspan="2">south</td></tr>
<tr><td>Situation as to the direction</td><td>ascending صاعد increasing زايد</td><td>descending هابط decreasing ناقص</td><td>descending هابط increasing زايد</td><td>ascending صاعد decreasing ناقص</td></tr>
<tr><td>Added to the horizon</td><td>Situation in the quadrants of the heaven</td><td>easterly زايد increasing مقبل rising</td><td>westerly ناقص decreasing زائل declining</td><td>southerly زايد increasing مقبل rising</td><td>northerly ناقص decreasing زائل declining</td></tr>
</table>

Figure 17 (p. 104 of text)

104:1 And if the different opinions and the confused operations in this respect are resolved,

2 the wisest thing is to determine the time when the planet advances its distance from

105:1 the earth, and the Hindus call it the modified hypotenuse.

2 And as an example, let ABJ (Figure 18) be the deferent with center D

3 and H the center of the universe and Ṭ the center of the equant. And let B

4 be the center of the epicycle, and K the position of the planet on it. And

5 KH will be this modified hypotenuse, and it is its (the planet's) distance from the earth. And because HB is the hypotenuse of a right triangle with

6 legs BZ, the sine of the unmodified center, and ZH the cosine of this

7 center, having added to it ṬH, the eccentricity, or diminished from it

8 as required by the situation, or (when it is) devoid of increase or decrease in a third, (when) the eccentricity is

9 partitioned for a fourth. So HB will be known, and ᶜK is

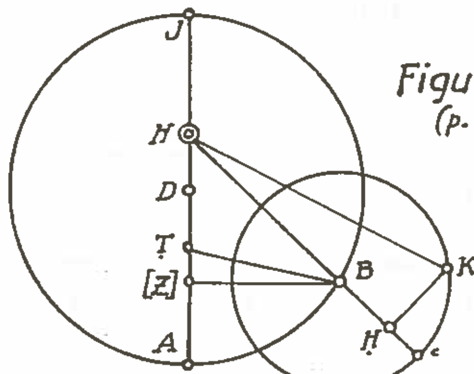


Figure 18 (p. 106 of text)

105:10 the modified anomaly. And its sine, KḤ, is known in units such that [B]ʿ is the total sine.
11 And if it is converted to the measure which we mentioned for the radius, Bʿ
12 becomes of the same sort as AD. And if ḤB is added to ḤB or subtracted
13 from it according to what is required by the situation, ḤH will be known, and KH, the required diameter
14 will be its hypotenuse, and that of KḤ. So it is known and its ratio to
15 sixty, the amount of the radius of the deferent, is as the ratio of the required, converted to these (units).
16 And if this is done to two planets, their situation with respect to the mean distance will be known as to
17 positive or negative elevation. And by comparing one of them with the other their transit will be determined as to whether it is in one path,

106:1 or whether one of them is elevated above the other, and the magnitude of the elevation, because what was done
2 is from one magnitude.
3 As for the latitude of the two planets, if they are equal in one direction, the
4 elevation between them will vanish due to its transit at one small circle (of latitude?), but if they differ
5 the elevation between them will occur then. They are in the condition of equality if one of them is
6 at the extreme of its latitude and the other increasing in latitude. And there is no doubt but that the one increasing is ready
7 for elevation. And if the one that is in excess were decreasing it is more liable for the contrary of

:1 elevation and its weakness. The preceding base (of computation) is not followed for it which makes the ratio of the latitude

107:2 of each one of them to the extreme of its latitude as the ratio of the required to one, so that
3 they would be transformed to one scale (for comparison). Such a procedure might give the highly elevated one as the depressed one.
4 As for two planets which are at the quadrants of the heaven with respect to the horizon,
5 if the ratio of the times (<u>azmān</u>) of each one of them from the degree of midheaven
6 to one hundred and eighty (is) as the ratio of the required to one, there will result the magnitude of their deviation from
7 the tenth (house). From the difference between them the magnitude of the elevation of one of them above the other is determined.
8 With all this, consideration of the basic rules
9 of the craft of astrology is relevant, but no complications arise which require explanation.
10 The book is finished, praise be unto God, the Lord of the worlds, and the blessings of God upon His Prophet
11 and His Messenger, Muḥammad, and his virtuous relatives.
12 And we finished copying it in Mosul (Mawṣil) in Dhū al-Qaʿda
13 in the year 631 A.H.
(July/August, 1234)

C O M M E N T A R Y

In this commentary references to the text and translation are made by pairs of numbers separated by a colon. The first number gives the page of the text, the second the line. References to the bibliography on page 187 are indicated by numbers enclosed in square brackets.

Concerning the life of al-Bīrūnī himself, the reader will find a wealth of biographical and bibliographical material in [7].

1:2-14. Introductory Definitions

In medieval astronomical Arabic the word mamarr, "crossing", has the standard technical meaning conveyed by the modern term "meridian transit". In this treatise Bīrūnī uses the same word in a number of more general senses, which we continue to translate by "transit". It is to the explanation of these usages that he has devoted this treatise. He begins by setting up three cosmic dimensions: length (or longitude), width (or latitude), and thickness.

The first appertains to displacements more or less east or west with respect to a terrestrial observer. We say more or less, because the usage comprehends not only motions entailed by the daily rotation, motions in right ascension, but also the slow displacements of the planets along the ecliptic, motions in longitude.

The second dimension is, roughly speaking, measured north and south. Again, however, displacements either in celestial latitude or in declination are included in this category.

The third dimension involves motions normal to both the first two, that is, along the radius vector from the earth's center to the celestial object in question. It is appropriate that the concept of thickness be associated with it, since it deals with the thickness of the hollow spherical shell of the ether.

With each of these dimensions one or more varieties of transit is associated. The succeeding passage deals with the transit in longitude.

2:2. The two motions here referred to are, respectively, the rapid rotation of the celestion sphere once per day from east to west overhead, and the much slower proper motions of the planets from west to east among the fixed stars and along the ecliptic.

2:17. Bīrūnī here draws a distinction between the daraja (degree) of a planet and its darajat al-mamarr (degree of transit). The former is the common medieval designation

for its celestial longitude; the latter is the point of intersection between the ecliptic and the perpendicular dropped from the star to the celestial equator.

3:4-12. The term "rays" here is an astrological one and refers to the influences which various zodiacal bodies were supposed to project back at other bodies in configuration with them.

Tasyīr (aphesis, directio) is likewise an astrological term, usually referring to the process by which the length of a person's life was supposedly predictable by associating it with a moving point on the ecliptic. This passage in the text, however, is unintelligible to us.

3:14 - 6:5. Associated Pairs of Zodiacal Signs

This is a standard part of astrological doctrine in which, however, nomenclature and definitions differed somewhat, as this passage shows.

The term mudkhal (introduction) was used as part of a standard title for a number of general treatises on astrology (kitāb al-mudkhal ilā ʿilm ṣanāʿat al-nujūm), for instance, the work attributed to Vettius Valens below, and the "Great Introduction" of Abū Maʿshar also mentioned below.

The Vizhīdhaks, as Nallino ([19], vol.v, p.239; vol. vi, p.291) has shown, are Pahlavi versions of Vettius Valens' "Anthology". The latter was an astrologer who flourished in the second century A.D., or thereabouts. His name went into Arabic as Wālīs (or Fālīs) al-Rūmī (cf. [17], p.376). Bīrūnī mentions the Vizhīdhak in others of his works, in the "India" ([4], transl., vol.i, p.158) and in the Tafhīm ([5], p.212) as al-Bizīdhaj al-Rūmī.

The first set of pairs of signs associates Gemini with Cancer, Taurus with Leo, and so on, called by Wright ([5], p.227) corresponding in course. (Cf. also Bouché-Leclerq [8], p.161). This simply couples pairs of signs, or ecliptic points, which are equidistant from a solsticial point. Such pairs enjoy the properties enumerated by Bīrūnī. The

ortive amplitude (siʿat al-mashriq) of a point on the celestial sphere is the distance along the local horizon from the east point to the place where the point in question crosses the horizon in rising.

4:5. The term madār in an astronomical context usually refers to any of the small circles on the celestial sphere having the north pole as pole. In the course of the daily rotation any point not on the equator traces out a madār.

4:7. This corresponds to Bīrūnī's dictum in the Tafhīm ([5], p.229.) The usage of Ptolemy is different; see the Tetrabiblos, [23a] i,14, and [8], p.163.

4:9 - 10. The reference here is to the varying angle at which the ecliptic cuts the eastern horizon in the course of the daily rotation. Right ascensions are those witnessed by an observer stationed on the terrestrial equator. For an observer north or south of the equator the ecliptic crosses the horizon more and more obliquely as the observer moves away from the equator.

4:12. As Bīrūnī indicates presently, in line 18, the associated pairs are now Aries with Pisces, Taurus with Aquarius, and so on, pairs equidistant from an equinoctial point. The term equipollent is used by Wright ([5], pp.226, 227; cf. Tetrabiblos [23a], i,15).

5:10. The Abū Maʿshar here referred to, and frequently in the sequel, is Jaʿfar ibn Muḥammad al-Balkhī (fl. 850) the paramount astrologer of the Middle Ages, and known in Europe as Albumasar. His Great Introduction (Kitāb al-mudkhal al-kabīr ilā ʿilm aḥkām al-nujūm) exists in the Arabic original and in Latin translations ([10], p.88).

5:14. This al-Saifi is reported by Bīrūnī ([27], p.85) to have written a work on an astronomical instrument. He is apparently otherwise unknown to the literature.

5:16. In astronomical writings the words falak (pl. aflāk) and mantaqa (pl. manātiq) are frequently used interchangeably, as falak al-burūj and mantaqat al-burūj for ecliptic. We translate them as heaven, or orbit, or circle, or sphere depending on the context.

6:16. The year-transfer (taḥwīl al-sina) is the instant of the vernal equinox. Its determination was a matter of great moment in astrology. (Cf. [5], p.320.)

6:19-7:3. A triplicity consists of a set of three zodiacal signs, equally spaced four signs apart on the ecliptic. There being twelve signs, it follows that there are four different triplicities. The mean motions of Saturn and Jupiter are of such magnitude that, roughly speaking, the former traverses eight signs while the latter is traversing twenty. This implies that when a mean conjunction occurs between these two planets, the next will take place about eight signs farther along, i.e. usually in the same triplicity. The mean advance is about three degrees more than eight signs, so that after about twelve conjunctions in one triplicity the point of mean conjunction pulls forward into the next triplicity. This phenomenon is the shift of transit. For a more detailed discussion of the same topic the reader may consult [13], p.259.

7:4-7:14. The general idea seems to be to define elevation with respect to the local horizon and in terms of the astrological "houses". At any given instant the ecliptic is divided into four unequal arcs by the following four points, known as centers, (or pivots, or cardines): the ascendant (or horoscope) and descendant are the ecliptic points then crossing the eastern and western horizon respectively. Upper and lower midheaven are the points in which the ecliptic intersects the local meridian. These four arcs are subdivided into three parts, and each of the resulting twelve arcs is a house. These are numbered in a direction opposite to the daily rotation, starting from the

one immediately below the ascendant. There is then some sense in calling a planet in the tenth or eleventh house elevated, since it is already in the upper part of the ecliptic, for the time being, and is still rising by virtue of the daily rotation.

7:15 - 8:5. This seems to be the same type of situation as indicated in the previous passage, except that now the origin is taken as another planet instead of the ascendant.

The tenth (house) of the tenth (house) is indeed the seventh, for the operation of finding the tenth can be regarded as a backward rotation through three houses, and two of these carry one from the first house to the seventh.

8:10-9:5. The object of this passage is to explain why, as we would put it, north is taken as positive in measuring latitudes, and of two celestial objects the one farther north is said to be elevated above the other. It is because the northern hemisphere is known, and known to be inhabited.

10:9-16. In Sanscrit these phenomena are called vyatīpāta and vaidhṛta; see [25], p.13.

10:17. Ibn al-Nadīm ([17], p.385) calls this individual Ibn al-Bāzyār as does Bīrūnī in 84:15 below. His book is called Kitāb al-qirānāt wa taḥwīl sinī al-ʿālam.

12:5-9. Bīrūnī is here making a point, to which he returns later, that since the maximum distances from the earth of some planets are exceeded by the minimum distances of others, a statement to the effect that such and such a planet passes over another is not to be taken as referring to their actual distances, but is an expression of a convention which implies something quite different.

13:5-6. The period of Saturn is about thirty years, that of a lunation is about thirty days.

13:7-8. The annual period of the sun is about twelve months; the period of Jupiter is about twelve years.

13:10-13. Thus the arrangement is, from the earth outward: the moon, the sun, with Venus rotating on an epicycle about it, then Mercury, Mars, Jupiter, and Saturn.

13:14-15. This arrangement, of having the inferior planets rotate about the sun, and which indeed corresponds to the facts, is due to Heraclides of Pontus (fl. 350 B.C., cf. [12], p.255).

13:18. This work is listed by Ibn al-Nadīm ([17], p.356) with the title *kitāb al-radd ʿalā Bruqlus*. Bīrūnī refers to this book in at least two other places, in his "India" ([4], transl., vol.i, pp.226, 231).

14:9-10. This is the ordinary association of each day of the week with a planet.

15:8-12. This is Bīrūnī's first mention, in this treatise, of the planetary "sectors", a topic to which he will revert frequently in the sequel. The reader will find a detailed discussion of the subject in [13], together with a table of numerical values.

It was customary to consider both the deferent and the epicycle as divided into four segments, each called a sector (*niṭāq*). These are illustrated in Figures 1 and 5 respectively, where the Roman numerals indicate the numbers attached to each sector. It will be noticed that the initial points of the first and third sectors are the apogee and perigee of the deferent and epicycle respectively. The initial points of the second and fourth sectors are always symmetrically disposed with respect to the line of apsides, but their definitions differ depending on the type of sector. In this passage Bīrūnī is dealing with "distance sectors", and point Ḥ in Figure 1, a deferent point at mean distance from H, the center of the universe, is the beginning of the second deferent distance sector. In like manner, in Figure 5, point Ṭ is one of two epicycle points at mean distance from the center of the universe, here Z. Ṭ is the beginning of the second epicyclic distance sector.

A point in sectors I and IV is at a distance greater than the mean, and these sectors are called "ascending" (*ṣāʿid*, in the sense of the "first opinion",see the note to 71:12 below). Sectors II and III mark positions of distance less than the mean and are called "descending"(*hābiṭ*, again according to the "first opinion").

It is also true that, as Bīrūnī remarks, in one of the two ascending sectors (I) the point is coming down, i.e. getting closer to the center of the universe, and in one of the descending sectors (III) it is ascending, i.e. receding from the center.

15:13 - 18:8. This passage is largely etymological. In several places Bīrūnī seems to make tacit application of the transition from *k* to *j* of words passing from Middle Persian to Arabic or modern Persian. Examples are *zīk* to *zīj*, and *vizhīdhak* to *bizīdhaj*. Thus he attempts to obtain *jūī* from *kūī*, and *auj* from *auk*.

He is motivated partly by a desire to explain why the Persians used the word *jawwī* (? or *javī*) to denote phenomena related to the deferent, that is the "heaven of the (deferent) apogee", while they employed the term *watar* (Arabic for *chord* or *cord*) for things related to the epicycle.

He derives the latter usage from the mythical cords or halters attached to the planets, which,pulled by deities seated in the heavens, provided a primitive Hindu explanation for the retrogradations of the planets (cf. [5], p.107.). Since the retrogradations are phenomena connected with the epicycle, hence the association of *watar* with the epicycle. He remarks incidentally the Persian word *zīj* which also means cord, and which eventually came to denote sets of astronomical tables. (Cf. [14], p.123.)

Reverting to jawwī, Bīrūnī alludes to the same word in the common Persian term for the "right sphere", javī (or jūyi) rāst, the equivalent of Arabic al-falak al-mustaqīm and Latin sphaera recta. The astronomical situation referred to is the appearance of the celestial sphere to an observer located on the terrestrial equator. Under these circumstances all points on the celestial sphere rise across the horizon at right angles, whence the modern term "right ascension".

As to the alleged derivation of javī from kūī, Professor R. N. Frye writes that although there is a Middle and New Persian gōy, "ball" or "sphere", Bīrūnī's word cannot come from it and must in fact be the Arabic jawwī, "atmosphere".

Bīrūnī makes a second attempt, likening the motion of the stars in the diurnal motion to the motion of objects carried by a flowing stream, Persian jūī.

17:5. The word as it appears in the printed text is undoubtedly a garbled version of the Greek cognate of apogee. As was customary, the pi has gone into Arabic fa'. The gamma of the original no doubt went into jīm, the kha' which appears being the result of a dot placed above instead of below the character.

Bīrūnī is right about auj having come from Sanscrit (from ucca, apex), but the Sanscrit form in turn seems to have been Greek in origin. (Cf. [21], p.29.)

17:8. Bīrūnī's sīkrā'ī is the Sanscrit shīghra, "fast".

17:10. In other words, in all cases the period of the deferent equation is longer than that of the epicyclic equation.

17:12. The Hindu lunar theory recognised only one equation in the moon's longitudinal motion.

17:15. The same author and book is mentioned by Bīrūnī

([1], 69:6) in connection with astrolabes. The individual is otherwise unknown to us.

17:17. Here again is a garbled transliteration, this time of the Greek cognate of perigee. Again a pi has gone into fa', and gamma into jīm.

17:19-18:1. On the basis of the printed text we infer that the third letter in both transliterations should be either a ḥa' in both or a jīm in both. A dot, either added below the letter in the first word, or deleted from the second, would restore the situation.

18:6. Here the transliteration of the Greek cognate of "epicycle" has come through unharmed, except that there should be two qāfs, one for each kappa.

18:12 - 20:18. The discussion and the accompanying figure are here straightforward, to our mind unnecessarily complicated by Bīrūnī's use of the parecliptic (al-mumaththal), a circle of finite radius, concentric and coplanar with the ecliptic. The Islamic astronomers apparently felt a need for some reference circle, or scale, on which to measure longitudes, and the ecliptic itself would not do, it seemingly being regarded as beyond the orbits of all the planets.

Here a celestial object travels with uniform speed along a circular deferent. The object is to obtain expressions for the points at which it has maximum, minimum, and mean distance from H, the center of the universe. If we call the mean longitude measured from apogee $\bar{\lambda}$ (in Arabic markaz al-kawkab, the "center" of the planet), the points of maximum and minimum distance are those at which $\bar{\lambda}$ is 0° and 180° respectively.

For a deferent AḤSZ lying wholly inside the parecliptic, of which only the arc AB is shown on the figure, Bīrūnī easily shows that Ḥ and Z are the points of mean distance from H. Corresponding values of $\bar{\lambda}$ are given by

arcs $A\dot{H}$ and $A\dot{H}\dot{S}Z$ respectively.

The true longitude, λ, also measured from the apogee, can be given in terms of arc length along the pareclipt ic. For instance, the value of λ when the object first reaches its mean distance position is the pareclipt ic arc AB.

Bīrūnī points out (19:18) that for mean distance

$$\bar{\lambda} = \angle ADH > 90^\circ > \angle AH\dot{H} = \lambda.$$

These relations remain invariant regardless of the size of the deferent with respect to that of the pareclipt ic. Two other deferents are drawn, one ᶜLJ lying wholly outside the pareclipt ic and with center M, the other KB, only partially outside the pareclipt ic, and having center $\dot{T}$. For all

$$\text{arc } {}^{c}L = \text{arc } KB = \text{arc } \dot{H}A = \bar{\lambda}$$

for the mean distance.

At all times the difference between $\bar{\lambda}$ and λ is e, the "equation" (al-taᶜdil). See 56:9 and Figure 10 below.

20:11-14. The Medieval Sine Function and the Maximum Equation.

The "sine" (al-jaib) here alluded to for the first time resembled the modern sine function in every respect save that in general the radius of the defining circle was other than unity. We distinguish between the modern and the medieval functions by using a capital initial for the latter. If the radius of the defining circle is R, the identical relation between the two functions is

$$\text{Sin } \theta \equiv R \sin \theta \equiv \text{Sin}_R\, \theta.$$

In some discussions, sines defined with respect to different radii will appear in the same expression. Where necessary we will avoid ambiguity by specifying the radius used for a particular sine by means of a subscript as shown in the third part of the identity above. We will denote an inverse (or arc sine) function by a superscript 1. Of course max Sin θ = Sin $90^\circ = R$, whence the term "total sine" (al-jaib kullhu = Latin sinus totus) or the "greatest sine" (al-jaib al-aᶜẓam).

It will be shown below (56:19) that for an object moving

as depicted in Figure 1 the maximum value of the equation is given by

$$e_{max} = \text{Sin}_R^{-1}\, d,$$

where d is the eccentricity, DH, and R is the radius of the deferent, $D\dot{H}$.

Bīrūnī now points out that the first deferent distance sector (cf. the comment to 15:8 above) is

$$\text{arc } A\dot{H} = 90^\circ + \text{Sin}^{-1}\overline{SH} = 90^\circ + \text{Sin}^{-1}\frac{d}{2}$$
$$= 90^\circ + \text{Sin}^{-1}(\tfrac{1}{2}\text{ Sin } e_{max}).$$

(See the comment to 41:9 below).

20:17. Here, as in 17:12, Bīrūnī evinces knowledge of the fact that for the pre-Ptolemaic Greeks, as well as for the Hindus, the motion of the moon was regarded as exhibiting only one periodic perturbation.

21:1. Kūshyār was an Iranian astronomer who flourished in the eleventh century (cf. [26], p.83). He was the author of Zījes 7 and 9 in [14]. In the Leiden copy of the Jāmiᶜ Zīj there is no table of sectors.

21:3. To show what Bīrūnī has in mind, note that, for $0 < \theta < \frac{\pi}{2}$,

$$\tfrac{1}{2}\sin 2\theta = \sin\theta\cos\theta < \sin\theta.$$

Put $2\theta = e_{max}$ to obtain

$$\tfrac{1}{2}\sin e_{max} < \sin\frac{e_{max}}{2},$$

and

$$\sin^{-1}(\tfrac{1}{2}\sin e_{max}) < \frac{e_{max}}{2}.$$

Here the left-hand side is Bīrūnī's rule as given above, and the right-hand side is the rule of Kūshyār and Abū Maᶜshar. Note that when e_{max} is small, i.e., for small eccentricities, the last expression above is an approximate equality.

21:6. The zīj of Abū Maᶜshar (63 in [14] see 5:10 above)is not extant.

21:7. Representation of Numbers

The reader must be prepared to encounter such transformations as 2;10,30 = 130½' = 130½ minutes = 7830", i.e. sexagesimals expressed as decimal integer multiples of the smallest fractions involved. The minute and second symbols need not imply angular measure.

22:4-29:12. Various Values for the Maximum Equations of the Sun and Moon

This passage contains an unprecendently rich collection of parameters for the solar and lunar theories of Greek, Hindu, and Islamic astronomy. Some values are well-known, others are found uniquely in this source. Additional information on the solar equation may be found in [15]. Here the "equation" of a planet is the difference between its mean and true longitude.

22:5. The Ptolemaic maximum solar equation of 2;23° given here is correct (cf. [23], ed. of Halma, vol.i, p.201). Concerning the criticism of Ptolemy's technique, Bīrūnī has a detailed analysis of solar observations performed up to his time in Treatise 6 of his Masudic Canon [6], which would be well worth extensive study. See also 23;6.

22:6. The Theonic Canon there referred to is the Handy Tables of Ptolemy [22], commented upon by Theon of Alexandria.

22:8. The Almagest value for the maximum lunar equation is in fact 5;1° ([23], ed. of Halma, vol.i, p.277). In 23:12 below, however, Bīrūnī credits Ptolemy with precisely this value, and Theon, i.e. the Ptolemaic Handy Tables [22], with 5;0°.

22:9. The three astronomers Yaḥyā, Khālid, and Sanad, are among the best known of the "Companions of the Verification" (Aṣḥāb al-Mumtaḥan) who, under the patronage of the Caliph al-Ma'mūn produced the celebrated Mumtaḥan Zīj, 51 in [14]. The latter two are also reported to have written zījes of their own, 96 and 97 in [14]. See 23:6.

The value of 1;47 here attributed to Yaḥyā is new to us. See 23:18 below. The Escorial version ([14], p.132) of the Mumtahan Zīj, which is, however, corrupt has the common value 1;59°.

Ibn Yūnis, author of the Ḥākimī Zīj (14 in [14]) attributes to the Mumtaḥan group, observing at Damascus, the value of 1;59,51° (see [11], p.56).

22:11-12. The joint value of Khālid and Sanad here reported as 1;59,54° is otherwise unknown to us. It is very close to the result given immediately above.

22:13. In both the extant versions of the zīj (or zījes) of Ḥabash al-Ḥāsib al-Marwazī (see [14], p.126) the maximum solar equation is 1;59° as given here. The three sons of Mūsā ibn Shākir ([14], p.135) carried out many observations, but their zījes are not extant.

22:14. The zīj of al-Battānī has been published. In it ([18], vol.ii, p.81) the maximum solar equation is indeed 1;59,10°, as reported here by Bīrūnī. The latter's own value, as reported in his zīj ([6], p.716) is 1;59,39,18°.

22:15-17. The zīj (73 in [14]) of Abū al-Wafā' is extant only in a fragment, if at all. The four observational results here attributed to him, 1;58,58°, 1:58,45°, 1;59,7°, and 1;59,2,20°, are new to us. In the zīj of al-Baghdādī (3 in [14]) the value of 1;59° is attributed to Abū al-Wafa'.

22:18-23:1. Al-Ṣaghānī (see [26], p.65) was best known as an instrument maker. He worked in Baghdad, c. 980, under the patronage of the Buyid dynasty. Of the two values attributed to him, 2;0,20° and 2;6,6°, the former was obtained also by Mufliḥ, a freedman of the Banū Amājūr (see [11], p.152; [14], p.125). For examples of different results obtained from the same data by computing with the chord function rather than the sine the reader may consult [15].

23:2. Ibn al-Aʿlam (author of zīj 70 in [14]), whose maximum solar equation was 2;0,10°, was undoubtedly an observer.

23:4-6. Sulaymān ibn ʿIṣma was the author of zīj X216 in [14]. His value of 1;55,2° is partially confirmed in 23:19. Concerning the criticism of Ptolemaic technique, see 22:5 above.

23:7. The name of al-Nasafī has been wrongly transliterated in [14], p.136, as al-Sanafī. He is otherwise unknown to us. His value is 2;27°.

23:12. See the comment to 22:8 above.

23:13-14. In his Sanjarī Zīj (27 in [14]) al-Khāzinī attributes to Ibn al-Aʿlam a maximum lunar equation of 4;51°. This is slightly different from the 4;53 here cited. See the comment to 23:2 above.

23:15. Al-Sarakhsī is mentioned in several other places in Bīrūnī's works as the author of Zīj 45 (in [14]). This is his first notice in this book of the famous Sindhind ([14], p.129) an Arabic translation of one of the Hindu siddhāntas, probably the Brahmasiddhānta of Brahmagupta.

23:17. According to Sachau (in [3], p.424), al-Jaihānī was a famous polyhistor, a wazīr to the Iranian Sāmānid dynasty in the beginning of the fourth century of the Hijra.

23:18. This confirms and makes completely secure the maximum solar equation of 1;47° attributed to Yaḥyā ibn abī Manṣūr, working under al-Ma'mūn, in 22:10 above.

23:19. We have $1;47°(1\frac{1}{14}) \approx 1;55°$, which is close to the value of Sulaymān given in 23:4 above.

Further, $1;47°(1\frac{1}{7}) \approx 2;2°$, for the Damascene value, but compare this with the comment to 22:9 above.

24:1. Now $1;47°(1\frac{2}{9}) \approx 2;11°$ for the Sindhind (cf. 23:15 above). This is attested later, in 24:17.

24:2. In fact $1;47°(1\frac{1}{4}) \approx 2;14°$, which is indeed the value appearing in the extant and published version [16] of al-Khwārizmī's zīj (21 in [14]), and further attested by Bīrūnī in another work ([15], p.118).

24:3. Now $1;47°(1\frac{1}{3}) \approx 2;23°$, which is the Almagest value (cf. 22:5 above).

24:7-9. The number 2;14° as a standard Hindu value for the maximum solar equation is found in many places, e.g. [9], p.156. On the other hand the number written out in the text, 4;50°, for the maximum lunar equation appears nowhere else, and we prefer to restore it as 4;5[6]°, a well-attested parameter, the six being a scribal omission.

The Zīj-i Shāh (30 in [14]) was translated into Arabic from a Pahlavi original. No copy is now extant, and the problem of reconstructing its contents and sources is one of great significance for the history of pre-Islamic Iranian astronomy. (Cf. [13]). Here Bīrūnī indicates that its contents were of Hindu origin.

24:11. The truth of this statement is fully demonstrated in the sequel. It will be seen that differences in Hindu parameters have nothing to do with differences in observations, but are the results of successive approximate computations in which different radii, for defining sines, and different approximations to π are used.

24:13-16. The rule is

$$\beta = \frac{9}{4}\,\mathrm{Sin}_{150'}\,\theta,$$

where β is the lunar latitude and θ is the argument of the latitude. Then indeed $\max\beta = 4;30°$, and

$$\frac{\max\beta}{\mathrm{Sin}\,90°} = \frac{\max\beta}{R} = \frac{9}{5}\,,$$

as Bīrūnī says. Both this max β and this R are standard

Hindu parameters.

24:17. Here the maximum solar equation of 2;11° is again ascribed to the Sindhind, as in 24:1 above. There seems little doubt but that the reference is here to an individual, Yasᶜa al-Ma'mūnī, otherwise completely unknown to us. It would be tempting to read into the text "the Ma'mūnic (zīj)" (cf. 23:18 above).

24:18. This is the first mention in this treatise of the early Islamic astronomer (or astronomers) named al-Fazārī, closely associated with the Sindhind Zīj. See the discussion in [15], p.119.

24:19-25:5. The beginning of this passage has been garbled in the text, but there is little doubt but that our restoration is valid. The two rules are, for the equation of the sun

$$(1 - \frac{1}{8}) \text{ Sin}_{150'}\bar{\lambda} = 2;11,15° \sin \bar{\lambda},$$

and for the equation of the moon

$$2 \text{ Sin}_{150'}\bar{\lambda} = 5° \sin \bar{\lambda},$$

where $\bar{\lambda}$ is the "argument", (al-ḥiṣṣa) the mean longitude measured from the apogee.

The parameter 2;11,15° is independently attested (in [15], p.119), also in connection with al-Fazārī and the Sindhind, but here with an R = 3270'. The maximum lunar equation of 5;0° is not far from the standard Hindu 4;56°, but it is precisely a Ptolemaic value (cf. 22:8 above).

An R of 150' is associated with al-Fazārī and with the Sindhind elsewhere, e.g. 31:5, and [2], 120:1. In the latter place the book is called the "Great Sindhind Zīj" (Zīj al-Sindhind al-Kabīr).

Bīrūnī's suggestion is to make the rule

$$(1 - \frac{1}{9}) \text{ Sin}_{150'}\bar{\lambda} = 2;13,20° \sin \theta.$$

This would indeed result in a maximum solar equation nearer to the 2;13° cited in the next passage.

We note that in all these expressions the "solution by

sines" is used to determine general values of the equation. (Cf. [15], p.118.)

25:6-8. This sentence makes two unrelated statements. The first is to the effect that in some versions of the Shāh Zīj the maximum solar equation is 2;13° rather than the more common 2;14° cited previously. This is of interest as indicating that we cannot count on a single, canonical text of this document.

The second part of the sentence becomes clear if we write

$$2(1 - \frac{1}{75}) \text{ Sin}_{150'}\bar{\lambda} = 4;56° \sin \bar{\lambda}$$

for the lunar equation and recall that 4;56° is the standard Hindu value for the maximum lunar equation. (See 24:7 above).

Note that this passage associates with the Shāh Zīj an R of 150'.

25:9-26:3. The Arabic-Persian term kardaja (pl. kardajāt) is usually derived from the Sanscrit kramajyā ([19],p.219). Originally it seems to have stood for a unit length of arc equal to one twenty-fourth of a quadrant, i.e. $3\frac{3}{4}°$ (see 25:16). Here it is a name for the variety of sine function being used.

The first rule given says the equation of the sun is

$$\frac{105}{2616} \text{ Sin}_{54;30}\bar{\lambda}.$$

The passage does not tell us the value of R shown, namely 54;30 = 3270', but the same rule is given in another work of Bīrūnī ([2], 133:6), and from it the R can be inferred. The 3270 is given explicitly in 27:2 below. In 27:1 and in the other source also these sines are called "kardajāt of the Sindhind", and in fact Bīrūnī says there that al-Fazārī gives this rule in the Sindhind Zīj (see [15], p.119).

We obtain the maximum equation implicit in this rule by putting $\bar{\lambda}$=90° to obtain

$$(\frac{105}{2616})(\frac{3270}{60}) = 2;11,15°.$$

precisely the number given in 25:1 above.

The second rule gives for the solar equation

$$\frac{7}{180}\,\mathrm{Sin}_{57;18}\bar{\lambda},$$

where now R is specifically stated to be 3438 = 57;18. This well-known parameter is associated with Āryabhaṭa (fl. 500 A.D., the first of two scientists with the name) and was adopted because if θ is small and measured in minutes of arc

$$\mathrm{Sin}_{57;18}\theta \approx \theta,$$

a property resembling that of the modern sine function for θ in radians.

Again putting $\bar{\lambda}=90°$ in the rule, to obtain the maximum, we have

$$\frac{7}{180}\left(\frac{3438}{60}\right) = 2;13,42°,$$

exactly as Bīrūnī says.

The third rule in this passage is

$$\frac{10}{107}\,\mathrm{Sin}_{54;30}\bar{\lambda}$$

for the lunar equation, from which the maximum equation is

$$\frac{10}{107}\left(\frac{3270}{60}\right) \approx 5;5,36°,$$

a number close to, but not identical with the five degrees attributed to al-Fazārī in 25:2 above.

The fourth and last rule gives the lunar equation as

$$\frac{10}{11[6]}\quad\mathrm{Sin}_{57;18}\bar{\lambda},$$

from which the maximum equation comes out as

$$\frac{10}{116}\left(\frac{3438}{60}\right) \approx 4;56,23°,$$

the number given in 26:2. The text has 117 in the denominator, but this requires emendation to 116 in order to yield the maximum equation shown, and moreover in the passage following this, where the same rule appears, also associated with the sines of Āryabhaṭa, the 116 is cited unmistakably, and more than once.

<u>26:4 - 26:19.</u> This passage is of interest as giving us a short excerpt from a work which is known to us only from a

mention in Bīrūnī's India ([4], ed., p.228; transl., vol.i, p.xxxiv; vol.ii, pp.52, 378). Author and date of the Harqan are unknown to us.

Since the quotation itself gives in verse form the rules for the equations of the sun and moon associated in the previous passage with the sines of Āryabhaṭa, namely

$$\frac{7}{180}\,\mathrm{Sin}_{57;18}\bar{\lambda} \quad \text{and} \quad \frac{10}{116}\,\mathrm{Sin}_{57;18}\bar{\lambda},$$

this demolishes Sachau's conjecture that the Harqan was a handbook for the conversion of Hindu, Arabic and Persian dates. It seems to have been a typical zīj. (See also [14], p.137.

The numbers needed are given as nonsense words made up of letters of the Arabic alphabet having the proper numerical value in the <u>abjad</u> system. They are ز = 7, (the text has the dot missing from the <u>za'</u> thus converting it into a <u>ra'</u>), ف=80, ق=100, و = 6, and ي=10, whence قف=180, and قيو=116. For the last number the text has, at 26:19, a medial <u>lām</u> instead of the correct <u>ya'</u>.

The term "elevate" (<u>rafᶜ</u>) indicates division by sixty, the sexagesimal base. Thus the "first elevate" (<u>marfūᶜ marra</u>) of 0;27 = 27' is 60(27') = (1,0)(0;27) = 27;0 = 27°.

<u>27:1-4.</u> Here is a categorical statement that the Sindhind was Brahmagupta's (fl. 650 A.D., author of [9]) Brahma-(sphuta)siddhānta, which is extant but unavailable in translation. (Cf. 25:10 above). In a letter Dr. Kripa Shankar Shukla, of Lucknow University, confirms that the R of the Brahmasphutasiddhānta is indeed 3270'.

We do not know what Bīrūnī means by "the mentioned operation". The maximum solar equation here given, 2;10,29°, will not be obtained by using Brahmagupta's R in the rule for the solar equation in the verse just above. This would give

$$\frac{7}{180}\left(\frac{3270}{60}\right) = 2;7,10°.$$

Professor B. L. van der Waerden has pointed out in a letter that one can infer from the dimensions of the solar epicycle in the Brahmasphutasiddhānta a maximum equation

quite close to the one of the text. From this

$$\sin e_{max} = \frac{13;40}{360} = \frac{41}{1080},$$

whence

$$e_{max} = 2;10,32^\circ.$$

There is a like difficulty with the maximum lunar equation of five degrees given in the same place. We note, however, that the rule

$$\frac{10}{109}\,\mathrm{Sin}_{3270'}\bar{\lambda}$$

will give the desired maximum of five, and the coefficient differs from that of the rule of 25:13-14 only in having 109 instead of 107 in the denominator.

<u>27:8-12.</u> According to Dr. K. S. Shukla, no scientist named Nābhāla has been encountered in the Sanscrit literature. His rule for the lunar equation is

$$\frac{31}{360}\,\mathrm{Sin}_{57;18}\bar{\lambda},$$

from which the maximum equation will be

$$\frac{31}{360}\left(\frac{3438}{60}\right) = 4;56,3^\circ$$

as stated in the text. There the name is written نبهل .

<u>27:13-17.</u> The Karaṇasāra was apparently translated into Arabic, but neither the original nor the translation is extant. Our only knowledge of it is from Bīrūnī's writings, but he mentions it frequently. References to it in his <u>India</u> [4] have been collected by Sachau in the notes to his translation of it, in vol.ii, p.156. The author was Vittisvara, son of Bhadatta (? Mihdatta), of Nāgarapura.

The rules for the solar and lunar equations are

$$\frac{10}{23}\,\mathrm{Sin}_5\bar{\lambda} \quad \text{and} \quad \mathrm{Sin}_5\bar{\lambda}$$

respectively, where now $R = 300' = 5$. The corresponding maximum solar equation is thus

$$\frac{10}{23}\left(\frac{300}{60}\right) = 2;10,26^\circ,$$

as stated. The maximum lunar equation is not given, but it is obviously five degrees.

<u>27:18 - 28:3.</u> The Karaṇatilaka likewise is known to us only through the writings of Bīrūnī. In his translation of the India ([4], vol.ii, p.306) Sachau has collected references to the Karaṇatilaka found in the former work. The author, Vijayanandin of Benares, however, is known independently of Bīrūnī, being mentioned, for instance, in [28],(p.54). Our passage strengthens Sachau's conjecture ([4], transl., vol.i, p.xxxvi, vol.ii, p.388) that an Arabic version of the Karaṇatilaka was made by one Abū Muḥammad al-Nāʼib al-Āmulī.

Be that as it may, the information that in the Karaṇatilaka $R = 200' = 3;20$ allows us to restore with confidence a gap in the text made by some copysist's lapse. The rules evidently were

$$\frac{2}{3}\,\mathrm{Sin}_{200'}\bar{\lambda} \quad \text{and} \quad \frac{3}{2}\,\mathrm{Sin}_{200'}\bar{\lambda}$$

for the solar and lunar equations respectively. From these the respective maximum equations are

$$\frac{2}{3}\left(\frac{200}{60}\right) = 2;13,20^\circ \quad \text{and} \quad \frac{3}{2}\left(\frac{200}{60}\right) = 5^\circ$$

as stated in the text.

<u>28:6-7.</u> This work and its author, probably Paulus Alexandrinus who lived in the fourth century or later, will be mentioned frequently in the sequel. The Pulisasiddhānta is one of the five siddhāntas of Hindu astronomy. (Cf. [4], transl., vol.i, p.153).

In the Hindu planetary theory the "equations", the periodic divergences between the true and mean planets, were accounted for by the use of epicycles rotating around a deferent of zero eccentricity. This was used for the equation due to the eccentricity as well as that of the anomaly. It was customary to specify these epicycles by giving their circumferences, measured in degrees of arc along the deferent. Thus the usage "circumference of the apogee" means the circumference of the epicycle used to produce the effect of the eccentricity. (Cf. [21],Appendix).

The circumference of the solar epicycle given here, fourteen, is the same as that of the Khaṇḍakhādyaka ([9],

p.xii). For the moon the epicycle circumference is thirty-one, again the same as that of the Khaṇḍakhādyaka.

28:8-10. Concerning the Brahmasiddhānta, see the comment to 27:1 above. These epicycle circumferences, for sun and moon respectively, are 13;40 and 31;26. [25] (p.52) confirms the solar value, but gives 31;36 for the moon. Our text value is confirmed in 29:7 below.

28:14-18. The approximation to the number π given here is quite good. It is

$$\frac{3927}{1250} = 3.1416 = 3;8,29,45,36,$$

is independently attested for Paulus ([4], transl. vol. i, p.168), and for Ya'qūb ibn Ṭāriq in the same place. It is used also by al-Khwārizmī in his algebra ([24], pp. 72 and 196).

By use of this approximation, Paulus' circumferences, and the relation $r = \frac{c}{2\pi}$, Bīrūnī now obtains the epicycle radii of the sun and moon respectively. They are

$$\frac{14}{2}\left(\frac{1250}{3927}\right) = 2;13,41^\circ$$

and

$$\frac{31}{2}\left(\frac{1250}{3927}\right) = 4;56,2^\circ$$

rounded off to two fractional places. These are Bīrūnī's results, except that for the moon he has apparently obtained his terminal 1 in the last place by truncating the next digit rather than by rounding off.

In line 18 here we restore the الجبر of the text to الجبر, which Bīrūnī uses in the sense of "rounding off".

28:19-29:4. Here we reverse the former process, that is to say, we infer the circumferences from the radii, where the latter are equated to the maximum equations. Bīrūnī sets up the ratio

$$\frac{c}{r} = \frac{c}{e_{max}} = \frac{360}{\mathcal{R}} = \frac{360}{57;18}.$$

The $\mathcal{R}$ = 57;18 = 3438', which we have encountered

before is attested in two places farther along in the text (32:1 and 55:2) as that of Paulus. Since, moreover, $\mathcal{R}$ is found by measuring the radius in minutes of arc along the circumgerence,

$$2\pi \approx \frac{360}{57;18} = \frac{6,0;0}{57;18} \approx 6;16,57,48$$

and

$$\pi \approx 3;8,28,54.$$

Taking now the standard Hindu maximum equations for sun and moon respectively, we have from

$$c = e_{max}\,(6;16,57,48),$$

$$(2;14)(6;16,58) \approx 14;2\ ,$$

and

$$(4;56)(6;16,58) \approx 30;59,41.$$

For the first Bīrūnī reports 14;3. The second is identical with our result.

29:5-12. Here is attributed to Brahmagupta the approximation

$$\sqrt{10} \approx 3.16228\ldots = 3;9,44,12,28,48,\ldots,$$

from $d^2 \approx c^2/10$. The same approximation is found in al-Khwārizmī's algebra [24].

Taking the maximum equations as being synonymous with the epicycle radii, Bīrūnī computes the latter, for the sun, as

$$\frac{c}{2\pi} \approx \frac{13;40}{2\sqrt{10}} = 2;9,39,\ldots$$

Bīrūnī's 2;9,9,40 may be the result of a copyist's error. Perhaps the passage should read "two parts and nine and two thirds of a minute".

In like manner, for the moon we have

$$\frac{31;26}{2\sqrt{10}} \approx 4;58,12,$$

which is precisely Bīrūnī's result. This confirms the moot reading of 31;26 for the lunar epicycle radius in 28:9 above.

Again reversing the process to obtain the epicycle circumferences in terms of the maximum equation we have for the sun

$$c = e_{max}(\frac{360}{R}) = 2;14(\frac{360}{54;30}) \approx 14;45.$$

which is Bīrūnī's result. Note, however, that while the R is Brahmagupta's (cf. comment to 27:1 above), the e_{max} is not the value near 2;10,30° attributed to him, but the standard Hindu parameter for the maximum solar equation.

The 14;45 differs considerably from the Khaṇḍakhādyaka and Brahmasiddhānta values for the same parameter given in the comments to 28:6 and 28:8 above.

For the moon we repeat the process to obtain

$$4;56(\frac{360}{54;30}) \approx 32;35,13,$$

which is close to Bīrūnī's [32];35,27. The restoration in the translation is of an obvious copyist's or typographical error. Again we have used the standard Hindu value for the maximum equation, not the five Bīrūnī attributes to Brahmagupta in 29:11. Again, moreover, the result differs markedly from the values of 31 and 31;36 Brahmagupta uses in the Khaṇḍakhadyaka and the Brahmasiddhānta respectively. (See the comments to 28:6 and 28:8 above).

29:13 - 33:14. Having disposed of the two luminaries, the text proceeds to a consideration of the maximum deferent equations of the planets. Again, while some of the parameters are from documents like the Almagest, which are available in modern editions, others are from sources which have disappeared.

As usual, the Ptolemaic Handy Tables [22] are referred to as the Canon, or zīj of Theon.

29:14-18. In the Almagest ([23] ed. of Halma, vol. ii, pp. 301-309) in the tables of planetary equations the first column (which we will denote by ③) after the columns of arguments, gives the equation of the deferent computed as though the epicycle center were on the equant. The next column (here denoted by ④) gives the correction to be added algebraically because of the fact that the epicycle is on the deferent, not the equant. The deferent equation is given by adding corresponding entries in the two

columns (i.e., ③ + ④). For practical computing it is better to add these once and for all and enter the results in a single column. This may have been done in the Handy Tables, and it looks as though in most cases Bīrūnī wrongly takes max ③ to be the Almagest maximum deferent equation. He should take max (③+④), which is probably found directly in the Handy Tables. The situation is seen from the table below.

Planets	Almagest max ③	max(③ + ④)
♄	6;32°	6;31°
♃	5;16	5;15
♂	11;32	11;25
♀	2;23	2;23(?)
☿	2;52	3;2

Upon comparing this with our text we note that for all Bīrūnī gives Handy Tables maxima which are identical with Almagest max(③ + ④). The conclusion is that the maximum deferent equations of the Almagest and the Handy Tables are actually the same.

30:1-3. This statement is probably correct, since we notice that in al-Battānī's Zīj ([18], vol. ii, pp.81, 129) for instance, the maximum solar equation is 1;59,10°, and the maximum deferent equation of Venus is 1;59°, doubtless rounded off from the former.

30:4-9. For the non-extant zīj of Ibn al-Aᶜlam (see the comment to 23:2 above) Bīrūnī gives maximum deferent equations of

♄	6;31° - 0;48° = 5;43°
♃	5;15 + 0;18 = 5;33
♂	11;25 + 0;0 = 11;25 (?)
♀	2;23 - 0;23 = 2;0
☿	3;2 + 0;38 = 3;40.

This set is not completely secure, since we cannot be certain as to the set which has been modified. In view of

the comment to 30:1 above, the value for Venus seems probable.

30:10 - 31:3. In the table below we consolidate the parameters given in this passage, adding for comparison appropriate sets from other sources. The first set consists of the maximum deferent equations Bīrūnī attributes to the "Hindus and Persians". The zījes of the Shāh and Abū Maᶜshar have been commented on in connection with 5:10,21:6 and 24:9. Yaᶜqūb ibn Ṭāriq (fl. 770, author of Zīj 71 in [14]), like al-Fazārī, was one of the early Muslim scientists engaged in putting Hindu and Iranian astronomy into Arabic. The second column is made up of the maximum equations of the center as excerpted from the published version of al-Khwārizmī's Zīj [16]. The third column gives the set attributed to the Shāh Zīj in the astrological work of Ibn Hibintā [12a], and in the last column appear the maximum equations of the center as used by al-Fazārī and obtained from our text.

	The Hindus and Persians	The published zīj of al-Khwār.	Zīj-i Shāh (Ibn Hibintā)	Al-Fazārī
♄	8;37°	8;36°	8;36,4°	8;37,30°
♃	5;6	5;6	5;5,49	5;[6]
♂	11;12	11;13	11;11,59	[1]1;10
♀	2;13	2;14	2;12,46	2;15
☿	4;0	4;2	4;[0,0]	4;[0]

Note that the 0;2° spoken of for Mercury in 30:17 actually appears in the zīj. The reference to Theon is doubtless to the fractional part of the 3;2 in the Ptolemaic maximum deferent equation of Mercury noted above.

The rules of al-Fazārī for finding general values of the deferent equation, presumably applied in a version of the Sindhind, are all of the form

$$k\ \mathrm{Sin}_{\mathcal{R}}\bar{\lambda} = \frac{e_{max}}{\mathcal{R}}\ \mathrm{Sin}_{150'}\bar{\lambda}\ ,$$

where $k = e_{max}/\mathcal{R}$ is a constant which depends on the particular planet, the rules are all examples of the "solution by sines" ([15], p. 119). $\mathcal{R}$ = 150' = 2;30 = 5/2 we have previously encountered associated with al-Fazārī (comment to 24:19) and the Sindhind (or the Great Sindhind) Zīj.

For Saturn the rule seems to imply

$$e_{max} = \mathcal{R}k = \frac{5}{2}\left(1 + \frac{1}{10} + \frac{1}{6}\cdot\frac{1}{10}\right)3.$$

But this leads to

$$e_{max} = 8;22;30°,$$

which is neither the result given in the text nor is it near the standard 8;36°. We note, however, that

$$\frac{5}{2}\left[1 + \frac{1}{10} + \frac{1}{2}\cdot\frac{1}{10}\right]3 = 8;37,30°,$$

which is what is called for. Hence the rule should probably be amended to read, "... the sine and its tenth and one half of its tenth..."

For Jupiter the rule gives

$$e_{max} = \mathcal{R}k = \frac{5}{2}\left[2\left(1+\frac{1}{5}\cdot\frac{1}{10}\right)\right] = 5;6°.$$

This is the standard value, and allows as to restore a waw (=6) in the text (31:7) following the ha'(=5).

For Mars the rule gives

$$e_{max} = \frac{5}{2}\left(1+\frac{1}{10} + \frac{1}{6}\cdot\frac{1}{10}\right)4 = 11;10°,$$

which is what the text has in 31:7, once a redundant dot has been removed from the alphabetical numeral.

With Venus,

$$e_{max} = \frac{5}{2}\left(1 - \frac{1}{10}\right) = 2;15°,$$

as in the text.

Finally, for Mercury

$$e_{max} = \frac{5}{2}\left(1 + \frac{3}{5}\right) = 4;0°,$$

which is what the text says, except that a typesetter has misread the Arabic sexagesimal zero symbol as a ha'in 31:8.

31:9 - 31:14. Concerning al-Sarakhsī see the comment to 23:15 above. We make his maximum deferent equations to be

♄	8;37°	, the Hindu value,
♃	5;15	, as in the Handy Tables,
♂	11;25	, as in the Handy Tables,
♀	2;24	, as in the Handy Tables plus a minute,
☿	4;2	, as with al-Khwārizmī.

The mention of cosmic days in obscure, but has something to do with the Hindu concept of a world-cycle marked by the conjunction of all planets near the vernal point. (See [14], p.131.)

<u>31:17-32:2.</u> Concerning Paulus and his Pulisasiddhānta, see the comment to 28:6 and to 28:19. Here Paulus' R of 3438' is attested. Bīrūnī again finds the circumferences of the epicycles by multiplying the maximum equations (regarded as radii) by an approximation to 2π determined by the particular R used. In the table below we list the maximum equations given in the text but converted into degrees, then the transformation into circumferences.

♄	9;28°	(568)(360)/3438 ≈ 59.5	≈ [60]
♃	4;44	(284)(360)/3438 ≈ 29.8	≈ 30
♂	11;16	(676)(360)/3438 ≈ 70.8	≈ 70
♀	2;14	(134)(360)/3438 ≈ 14.09	≈ 14
☿	4;28	(268)(360)/3438 ≈ 28.1	≈ 28.

The last column gives the circumferences as reported in the text, except that the one for Saturn does not appear. All these are identical with the corresponding parameters of the Sūryasiddhānta (of Varāhamihira), the Pançasiddhāntika, and the Paulisatantra (cf. [9], p. 48), except for Venus. For the latter planet these other documents have a circumference of 32. However, our text's 30 is further confirmed in 28:14 below.

These are the <u>manda</u> epicycles, those which account for the equation generated by the deferent in the Ptolemaic theory.

<u>32:4 - 12.</u> Some rules of the Karaṇatilaka are given in 27:18 - 28:3 above. This passage goes on to supply analogous rules for the planets, all presumably for the "solution by sines". General values for the deferent equations are given by the expressions

$$k \ \mathrm{Sin}_R \bar{\lambda} = \frac{e_{max}}{R} \ \mathrm{Sin}_R \bar{\lambda} = e_{max}\left(\frac{3}{10}\right) \ \mathrm{Sin}_{200} \bar{\lambda},$$

where k depends on the planet and R = 200' = 10/3 is the

total sine of Vijayanandin (see 28:1 above).

To infer the maximum equations in terms of the rules we have, for Saturn

$$e_{max} = Rk = \frac{10}{3}\left[\frac{3}{2}\left(1 + \frac{1}{6}\right)\right] = 5;10°.$$

This is far from any probable value, and the result in the text is no help. It has هن (<u>ha</u>' = 5 and a joined <u>nūn</u> = 50) which makes no sense as a sexagesimal. We restore it as [ى]• = 5;10, but both the rule and the result are probably garbled.

For Jupiter the rule is

$$e_{max} = \frac{10}{3}\left[\frac{3}{2}\left(1 + \frac{1}{6(10)}\right)\right] = \frac{61}{12} = 5;5°,$$

a good result, and which checks with the text.

For Mars,

$$e_{max} = \frac{10}{3}\left[3\left(1 + \frac{1}{7}\right)\right] = \frac{80}{7} \approx 11;25,42°,$$

which, if the seconds are truncated, is the text value.

For Venus,

$$e_{max} = \frac{10}{3}\left[\left(1 + \frac{1}{6}\right)\frac{1}{2}\right] = \frac{35}{18} = 1;56,40°,$$

which is the result given in the text if we convert the printed <u>ya</u>' (= 10) into a <u>nūn</u> (= 50) by changing some dots.

For Mercury

$$e_{max} = \frac{10}{3}\left[\frac{3}{2}\left(1 - \frac{1}{10}\right)\right] = 4;30°,$$

as in the text.

<u>32:12-14.</u> In this passage, as in 28:14-18, we convert from the epicycle circumferences of 32:1 to radii, using the approximation $\pi \approx 3927/1250$ associated with Paulus in 28:14.

Thus, for Saturn,

$$r = \frac{c}{2\pi} = 60\left(\frac{1250}{2(3927)}\right) = 9;33,$$

as in the text.

For Jupiter

$$r = 30\left(\frac{1250}{2(3927)}\right) = 4;46,30,$$

as in the text.

For Mars

$$r = 70\left(\frac{1250}{2(3927)}\right) = 11;8,30.$$

The text has 11,30, to which we have restored the medial digit.

For Venus

$$r = 14\left(\frac{1250}{2(3927)}\right) = 2;13,42.$$

The medial digit in the text is ل (=30), for يج (=13), a natural error for a copyist or typesetter ignorant of the conventions of the Arabic sexagesimal numerals.

Finally, for Mercury

$$r = 28\left(\frac{1250}{2(3927)}\right) = 4;27,24,$$

as in the text.

32:15-33:3. This passage gives a set of rules from the zīj al-Arkand ([14], p.138) for obtaining the maximum planetary deferent equations from 2;14°, the maximum solar equation. In the published translation of the Khaṇḍakhādyaka ([9], p.48) a set appears which resembles but is not identical with ours.

For Saturn our text would give a maximum equation of

$$2;14\left[4\left(1+\frac{1}{7}\right)\right] \approx 10;14°,$$

but this is far larger than any Saturn parameter reported, and we prefer to restore a "half" as in the translation to give the same rule as [9], namely

$$2;14\left[4\left(1+\frac{1}{2}\cdot\frac{1}{7}\right)\right] \approx 9;34°,$$

very close to Paulus' 9;33° in 32:13 above, but different from the standard 8;37°.

For Jupiter Bīrūnī has

$$2;14\left[2+\frac{1}{7}\right] \approx 4;47°,$$

which is not close to the standard 5;6°, but very near to Paulus' 4;46,30°, hence it is probably accurately transmitted. In [9], however the rule is

$$2;14\left[2\left(1+\frac{1}{7}\right)\right] \approx 5;7°.$$

For Mars our rule is the same as that of [9], namely

$$2;14[5] = 11;10°.$$

which is close to the value of Paulus and to the standard one.

For Venus both sets of rules put its maximum equation equal to that of the sun, 2;14°, also close to Paulus' result and to the usual Hindu value.

For Mercury both sets of rules prescribe the double of the solar equation, 4;28°. This is indeed near to Paulus' 4;27,24, but less close to the usual 4;0°.

33:4-10. This Awlath is otherwise unknown to us. He evidently made use of a table of solar equations instead of a table of sines to compute planetary equations. The rule is

$$e_p(\bar{\lambda}) = e_{\odot}(\bar{\lambda})\left[\frac{c_p}{14}\right]$$

where $\bar{\lambda}$ is the mean longitude of the planet reckoned from the apogee, e is the equation, c the circumference of the "apogee epicycle", and subscripts $\odot$ and p are used to mean solar and planetary respectively. We note first that

$$2\pi\,(\max e_{\odot}) = c_{\odot} \approx 14,$$

very nearly, and

$$c_p = 2\pi(\max e_p).$$

Moreover $e_{\odot} = \max e_{\odot} \sin\bar{\lambda} = \frac{14}{2\pi}\sin\bar{\lambda}$ in the "solution by sines".

Then the rule becomes

$$e_p(\bar{\lambda}) = \frac{c_p}{14}\,e(\bar{\lambda}) = \frac{2\pi(\max e_p)}{14}\left(\frac{14}{2\pi}\sin\bar{\lambda}\right) = \max e_p \sin\bar{\lambda}$$

which is indeed the "solution by sines" for the planetary equation of the center.

Bīrūnī is right in stating that choices of 54, 32, and 25 for the epicycle circumferences of Saturn, Jupiter, and Mercury respectively would yield maximum equations closer to the standard Hindu values than do 60, 30, and 28. We have already remarked that in [9] (p.48) the circumference for Jupiter is given as 32.

33:10 - 34:11. Having exhausted the topic of maximum deferent equations, Bīrūnī does not forthwith examine like sets

of parameters for the epicyclic equations. He first discusses mean distance positions for the moon, Mercury, and the other planets, in this order. The classification is forced upon him by the fact that he uses the Ptolemaic models for all, and there are significant differences between Ptolemy's mechanisms for producing the motion of the moon and Mercury, as distinguished from the other planets, the models of which differ from each other only in the numerical values of their parameters. In all three cases the argument and the figures are clear, and no extensive explanation is called for.

This passage deals with mean distance positions of the lunar epicycle. The reader unfamiliar with Ptolemy's model for the moon's motion will find it useful at this point to consult [20](pp.192-196). The essence of Bīrūnī's remarks may be restated concisely by noting that (in Figure 2) the locus of mean distances for the epicycle center is the circle JBḤL whose center H is the center of the universe and whose radius is the deferent radius. The locus of the deferent center is the smaller concentric circle ZDṬ. At any instant when the deferent center is at the general position Ṭ, a mean distance position for the epicycle center is L, the intersection with circle JBḤL of LM, the perpendicular bisector of TḤ.

34:12 - 38:2. This passage has to do with the determination of mean distance positions for the epicycle of Mercury. The Ptolemaic model for Mercury's motion is described in [20](p.200), for example. It will be recalled that for this planet the deferent center is carried about a small circle, Ṭ$^{\text{c}}$S in Figure 3, which passes through the equant center. As in the case of the moon, the locus of mean distance positions for the epicycle center is a circle, here BḤLJY, having its radius equal to that of the deferent and its center at the center of the universe, H. For any position of the deferent center, successively Ṭ, $^{\text{c}}$, and S, in the figure, consider the perpendicular bisector of the segment joining it to the center of the universe. The intersections,

here Ḥ, L, and Y respectively, of these perpendicular bisectors with the mean distance locus, give mean distance positions of the epicycle center.

38:3 - 41:2. This final passage of the section on mean distances deals with all the planets other than Mercury. The reader desiring an explanation of the Ptolemaic planetary model may consult [20](pp.198-200). Again (on Figure 4) the mean distance locus is a circle (LDṢ$^{\text{c}}$B) with radius equal to that of the deferent (ADḤB) and center at the center of the universe (H). When the epicycle center is at the apogee (A), mean positions of the planet occur at S and M; when it is at perigee, mean planetary positions are at Ṣ and $^{\text{c}}$. A mean position of the epicycle center is at B. For this situation mean planetary positions will be at N and W. A general position of the epicycle is not shown.

41:2 - 43:15. Bīrūnī now commences the consideration of maximum epicyclic equations and the related topic of epicycle sectors (cf. 15:8 above). He begins with the simple case of an epicycle carried at constant speed around a non-eccentric deferent. Under these circumstances maximum equation occurs when the planet is at Ḥ (in Figure 5), the radius vector ZḤ from the center of the universe to the planet then being tangent to the epicycle. The planet will be at mean distance from Z whenever it reaches a point of intersection between the deferent and the epicycle, B for example. The <u>first epicycle distance sector</u> will then be arc KDB, and the author's present objective is to obtain an expression for it in terms of the maximum equation, arc AB = e_{max}. This done, the boundaries of the other sectors, indicated by Roman numerals on the figure, follow immediately. In pursuit of this aim he directs (41:9) the dropping of perpendiculars ḤS and BM on KJ. In the case of the latter it would have been better to have said join B to M, the midpoint of AS. For it is the equality of SM and MA which is used in asserting that

$$\overline{AB}^2 = \overline{\d{H}A}^2 = \overline{ZA}.\overline{AS} = 2\overline{ZA}.\tfrac{1}{2}\,\overline{SA} = \overline{AM}.\overline{AB}.$$

From this follows the similarity of triangles BAM and JAB, and, JAB being a right triangle, so also is BAM. Hence BM is perpendicular to AS. Since

$$\frac{ZA}{AB(=A\d{H})} = \frac{AB}{AS},$$

the triangles BAS and BAZ are similar, whence, ZB being equal to AZ, AB = BS.

From this,

$$\frac{AS}{A\d{H}} = \frac{r}{R} = \frac{2\overline{AM}}{r}, \text{ or } AM = \frac{r^2}{2R},$$

where r and R are the radii of the epicycle and deferent respectively. So the magnitude of the first epicycle distance sector is

$$90^\circ + \widehat{DB} = 90^\circ + \mathrm{Sin}_r^{-1}\,\overline{AM} = 90^\circ + \mathrm{Sin}_R^{-1}\left(\frac{R}{r}\,\overline{AM}\right) = 90^\circ + \mathrm{Sin}_R^{-1}\left(\frac{r}{2}\right)$$
$$= 90^\circ + \sin^{-1}\left(\tfrac{1}{2}\sin e_{max}\right),$$

since for the epicyclic equation $\mathrm{Sin}_R\, e_{max} = r$.

The resemblance of this expression to that from 20:11 is not surprising in view of the fact that an eccentric deferent model with no epicycle is easily shown to be equivalent to this arrangement. The rule as given by Bīrūnī in 43:1-2 is valid provided that the sines there mentioned are with parameter r.

He is at pains to point out that since M is the midpoint of AS, B cannot be the midpoint of D$\d{H}$, or, as we would put it

$$\sin\tfrac{1}{2}\theta \neq \tfrac{1}{2}\sin\theta.$$

This is the crux of his criticism of Abū Maʿshar's rule, just as in the case of the deferent equation in 21:3 above. In this case the criticism is more valid, since for many planets the epicycle radius is a large fraction of the deferent radius. The approximation $\frac{1}{2}\sin\theta \approx \sin\frac{1}{2}\theta$ then deteriorates.

In modern symbols Abū Maʿshar's rule (43:6) is

$$90^\circ + \mathrm{Sin}^{-1}\left(\frac{(\mathrm{Sin}\ r)^2}{2}\right).$$

Note the resemblance in the argument of the inverse function

to our expression above for AM.

In 43:10-12 Bīrūnī rightly points out that AJ/BM ≠ BM/MA, rather AJ/AB = AB/AM. If the first expression were an equality we would have

$$MA = \frac{\overline{BM}^2}{AJ} = \frac{\overline{BM}^2}{2R}.$$

i.e., BM seems to be what Abū Maʿshar means by "Sin r".

<u>43:16 - 44:16.</u> In this passage the general Ptolemaic model is assumed, that is to say, the center of the deferent (in Figure 6) is displaced from H, the center of the universe. Two extremal positions of the epicycle are considered, those at maximum and at minimum distance from the center of the universe. For each of these Bīrūnī shows that the first epicyclic distance sectors, arcs KDB and Y$\d{S}$T respectively, are computable in terms of the parameters of the particular planet, the eccentricity and epicycle radius. His explanation is straightforward and requires no comment.

We note, however, that he gives to the arcs ʿB and T$\d{S}$ the name "mean depression" and "mean elevation" respectively, they being the increase or decrease in the size of the first sector caused by the eccentricity of the deferent. Bīrūnī points out that only at these two special epicycle positions is the corresponding mean distance position on the other side of the epicycle symmetrically placed.

<u>45:1-47:12.</u> This passage discusses the determination of the first epicycle distance sector for a completely general position of the epicycle on the deferent.

In 46:2 we have restored المرتبة of the text, which makes no sense, to المرئية, the masculine form of which occurs in the next line.

Assuming as given the "mean center" of the planet (the mean longitude measured from the apogee, angle A$\d{T}$B in Figure 7), Bīrūnī shows how to compute KʿD, the first epicycle distance sector, in terms of the epicycle radius and the eccentricity.

47:13 - 51:18. In this last passage dealing with the epicyclic distance sectors Bīrūnī concerns himself with the relation between the sector boundary and the intersection of the epicycle and the deferent for various positions of the former. Much of what he says seems obscure, and parts of the text may be garbled. Apparently he is working toward a quick, approximate method of finding the initial points of the sectors, for a general position of the epicycle, which involves less labor than the direct computation outlined in the preceding passage.

The entire discussion, from 33:10 to 51:18 seems artificial and somewhat pointless, since it is never applied, either in this text or anywhere else, to our knowledge. It seems reasonable to conjecture that the doctrine of "transits" and "elevations" was a holdover from some earlier and more primitive scientific milieu in which eccentric orbits and equants were unknown. If this was the case it is not surprising that Bīrūnī should have had difficulty in attempting to apply to it the complete Ptolemaic planetary apparatus.

He claims that the maximum "depression" (AL in Figure 8) of the sector boundary below the deferent will occur, not when the epicycle center is on the apogee, but when the epicycle is so located that a mean distance position on it coincides with L. The latter point is the intersection (on the apogee side) between the locus of mean distance positions and the line of apsides. Bīrūnī describes how to compute this "total depression"(48:11, al-inḥiṭāṭ al-kullī) in a manner analogous to the determination for the other cases. The usage is analogous to "total sine"(sinus totus, al-jaib al-kullī) for the sine of ninety degrees.

Bīrūnī next remarks (48:11) that when the epicycle center is at F the depression will be zero.

In like fashion he shows how to compute the "total elevation" arc TP. In 49:8 we have made a conjectured restoration. A little tampering makes some sense of the passage, but not much. Let the epicycle diameter be d and use the fact that radius KF is perpendicular to ZK. Since

PF is the maximum epicyclic equation for zero eccentricity, for small d,

$$\mathrm{PF} = \mathrm{Sin}^{-1} \frac{d}{2} \approx 2\ \mathrm{Sin}^{-1} \frac{d}{4}.$$

This presumably is the "arc of the chord" (50:3 and 50:17). But this is the sort of thing for which Bīrūnī so severely criticizes Abū Maʿshar.

In 50:8 the reference is probably to the interpolation function computed by Ptolemy to modify a planetary equation computed for an extremal (or mean) earth-planet distance to take account of the fact that its actual distance is somewhere in between the extremals.

The rule of 51:13 seems to say that for a general situation the elevation h (or depression) is

$$h = \frac{\delta}{\Delta}(\max h),$$

where δ is the deferent arc from the epicycle center to the epicycle position for maximum elevation, and Δ is the deferent arc between the epicycle positions of maximum elevation and zero elevation.

51:19-52:9. Bīrūnī now gives one of the two sets of parameters which would be needed to carry out the calculations indicated in the preceding sections, the deferent eccentricities. They are shown in the first column of our table.

	Eccentricities		
	Text	Almagest	Epicycle Radii
♄	3;34	3;25	6;30
♃	2;41,30	2;45	11;30
♂	6;33,30	6;0	39;30
♀	1;15	1;15	43;10
☿	3 to 9	3 to 9	2[2];30

He claims they are from the Almagest [23], but the actual Almagest eccentricities differ from them for all three superior planets. We are at a loss to explain these discrepancies.

52:10 - 53:2. Here Bīrūnī explains the variable eccentricity Ptolemy worked out for Mercury. (Cf. [20], p.200.) The deferent center is carried about on the small circle shown in Figure 9. In all circumstances the eccentricity is the distance from the center of the universe (H) to the deferent center. A general position of the latter is shown at Z, and Bīrūnī explains how to determine HZ. Since KD = DṬ = ṬH = 3, the extremal eccentricities will be 3 (= HṬ) and 9 (= HK) as given in the text.

53:3-6. This passage gives the second set of parameters required, the epicycle radii. The numbers in the text are shown in the third column of the table above. All are, as Bīrūnī says, from the Almagest, provided we restore the value for Mercury as shown.

53:6-55:13. This is a section of great interest in that it gives several sets of maximum epicyclic equations, some mentioned nowhere else in the literature. As Bīrūnī remarks, these follow from the magnitudes of the epicycle radii, as has been demonstrated in the comment on 41:2 above. The first column of the table below is the set Bīrūnī attributes to Theon's Canon, no doubt the Ptolemaic Handy Tables. They are the same as those found in the Almagest ([23], ed. of Halma, vol.ii, pp.301-309) except for that of Venus, for which the Almagest has 45;57°. The corresponding values in al-Battānī's zīj [18] are identical with those of our text.

	Theon's Canon	Ibn al-Aᶜlam	Shāh Zīj	Paulus: Shīghra epicycle circumference: Khand. & S-s.	Paulus: Shīghra epicycle circumference: Text	Paulus: Epicyclic e_{max}: Text	Paulus: Epicyclic e_{max}: Computed
♄	6;13°	5;48°	5;44° $\left(\begin{cases}+0;0,8\\-0;1\end{cases}\right)$	40	40	6;22°	6;22°
♃	11;3	11;3	10;52	72	72	11;32	11;28
♂	41;9	41;9	41;30 (-0;1)	234	255	40;32	40;35
♀	45;59	46;8	47;11 (-0;1)	260	290	45;15	46;10
☿	22;2	22;22	21;30 (-0;0,30)	132	135	21;36	21;29

Concerning Ibn al-Aᶜlam see 23:2 above. His maximum epicyclic equations make up the second column in our table.

Other parameters of the Shāh Zīj have been listed in the comment to 30:10 - 31:8. In the third column of the present table we list the maximum epicyclic equations as given in this passage, with the variants Bīrūnī notes entered in parentheses. To this zīj, to Abū Maᶜshar, and to al-Khwārizmī we will revert in connection with the tables on pp. 87 and 91 of the text.

Yaᶜqūb ibn Ṭāriq is mentioned in 30:11 above only to state that his maximum planetary deferent equations are those of the Shāh Zīj. Apparently two of his maximum epicyclic equations differ from those of the Shāh; for Jupiter he has 10;30°, and for Venus 46;16°.

Al-Sarakhsī is mentioned in 23:15 above.

For Paulus our text gives the circumferences of the shīghra epicycles (cf. 31:17 above). These are displayed in the fifth column of our table. For comparison we give in the preceding column the circumferences common to the Khandakhādyaka and the Sūryasiddhānta of Varāhamihira ([9], p.xii). The text also gives corresponding maximum equations, with the now familiar method (see the comment to 31:17 above)

$$e_{max} = r = \frac{57;18}{360} c = \frac{57;18}{6,0} c = 0;9,33\, c$$

of converting between them. The last two columns of the table give the text values for e_{max} and the results of our computation. It will be noted that, except for Venus, the correspondence is close.

We can only regret that Bīrūnī has been unable to give us the shīghra as well as the manda equations for other Sanscrit sources also.

55:14-58:10. Bīrūnī here reverts to the subject of sectors. He reminds us that he has already dealt with the (mean) distance sectors (see 15:8 and the commentary above), and now seeks to define analogous entities based on the angular velocity of the planet. Figure 10 tacitly assumes a

situation equivalent to the solar motion in Ptolemy's planetary theory, an object moving at constant speed in a circular orbit ABJD, the center of which (Z) is slightly displaced from the observer's station at H. $\bar{\lambda}$, λ, and e, have the meanings defined in the commentary to 18:12 above. He correctly asserts (in 56:2) that the angular velocity of the object as viewed from H will have a minimum at the apogee (A) and a maximum at perigee (J). At the same points the equation (e) will be zero. Further, the end-points, B and D, of the chord perpendicular to AJ through H are the points at which the angular velocity of the object attains its mean value (56:4). At the same positions the equation (e) is maximum and minimum. Put in modern terms, if e is regarded as a function of time, then, using the customary dot notation for derivatives with respect to time, since $\lambda = \bar{\lambda} - e$, $\dot{\lambda} = \dot{\bar{\lambda}} - \dot{e}$. Then, when $\dot{e}=0$ two simultaneous consequences follow: (1) $\dot{\lambda} = \dot{\bar{\lambda}}$, and (2) e has a maximum or a minimum.

These facts (or their equivalent) were well-known to the Islamic astronomers, and they are set forth in the Almagest. Nevertheless Bīrūnī takes time to show carefully (56:9-58:2) that the maximum equation does occur at B.

The four points A, B, J, and D are made the initial points of the first, second, third, and fourth deferent velocity sectors respectively (56:14).

In the Ptolemaic theory the sun is the only celestial object to which this model was applied without modification. For the planets, Ptolemy found it necessary to assume that the center of the epicycle moves on the deferent in such fashion that its angular velocity as viewed from Ṭ, rather than from Z, is constant. Ṭ is the equant center, so placed on the line of apsides that ṬZ = ZH.

Under these circumstances $\bar{\lambda}$, the mean longitude, will not be measured as shown on the figure, but by the angle AṬH. Bīrūnī claims (58:7) that the point giving maximum equation will still be B, but this is wrong. In fact, with the equant model, maximum equation will occur a quadrant's distance along the deferent from A. (Cf. [13], p.250).

58:11 - 59:9. Here Bīrūnī tacitly makes use of the facts applied above, namely that maximum (or minimum) equation implies mean angular velocity. Now the application is to the epicycle rather than to the deferent. If a simple, non-eccentric model is used, a tangent (ZD in Figure 11) to the epicycle from the deferent center marks the extremal epicyclic equation, angle ʿZD. Hence when the planet, in its course around the epicycle reaches D, its angular velocity as viewed from D will attain its mean value. Bīrūnī points out (59:3) that the point S in the epicycle of Figure 11 corresponds to H in Figure 10.

With the full Ptolemaic model the center of the universe is displaced to the eccentric position H. Then the maximum epicyclic equation will be angle LHK, LH having been drawn tangent to the epicycle. The arc KʿL then gives the magnitude of the first "adjusted" epicyclic velocity sector, while arc ʿḤD is that of the first (or fourth) "mean" epicyclic velocity sector.

59:9 - 60:6. Here Bīrūnī classifies the sectors as has already been done in the commentary to 15:8. Sectors I and IV are "ascending" (ṣaʿid), II and III are "descending" (hābiṭ).

In 60:2 we restore the sīn of the text's asʿad to a ṣād, making the word aṣʿad.

The fact that ṣāʿid may mean either "higher" or "ascending" may explain the ambiguity in terminology.

60:7 - 61:18. Here Bīrūnī announces a preference for the (mean) distance sectors rather than for the velocity sectors just defined. His reason for the preference is not clear to us.

He remarks that since for the planets the direction of rotation in the epicycle and in the deferent are the same, the maximum angular velocity (disregarding the effect of the equant) will occur when the planet is at epicyclic apogee. This situation is reversed with the Ptolemaic moon, since the direction of rotation in the lunar epicycle

is opposite that in the deferent.

With the planets the effect of the epicycle is so marked as to reverse the direction of advance of the planet as viewed from the earth, causing it for a time to retrograde. The instants at which the angular velocity vanishes, either in passing from direct to retrograde motion, or from retrograde to direct, are known as stations. Bīrūnī suggests (61:7) that the corresponding points on the epicycles might better be taken as sector boundaries instead of the points of mean angular velocity.

62:2-6. This passage is a graceful tribute from one great scientist to a greater, and to which we can only add a fervent amen.

62:9-14. Bīrūnī's rule is here

$$90^\circ + \mathrm{Sin}^{-1}(\tfrac{1}{2}\mathrm{Sin}\, e_{max})$$

as in the comment to 20:11. By "first opinion" he designates the distance sectors, as distinguished from the velocity sectors. The "unmodified argument" is of course $\bar{\lambda}$.

62:15 - 63:2. The rule for the end of the first velocity sector, the "second opinion", is

$$\bar{\lambda} = 90^\circ + e_{max}.$$

As Bīrūnī says, the same point is marked by $\lambda = 90^\circ$, angle AHB in Figure 10.

63:3-10. Here Bīrūnī mentions the manner of treating sectors in a number of zījes. All the tables of sectors available to us have been discussed in [13].

In neither of the extant versions of the zīj of Ḥabash ([14], pp.152,3) are there any sector tables. The same is true of the single version of Kūshyār's zīj (9 in [14]) examined by us. The summaries referred to in the passage are not extant.

63:10-13. Concerning the planetary deferent sectors see [13], pp.249-251, and the comment to 55:14 above.

63:14-17. The statement that only epicycle sectors are used for the moon supports our theory that the doctrine of sectors was developed in connection with a body of astronomical theory in which as in the Hipparchian and Hindu astronomy, only one lunar equation was dealt with(cf.47:13 above).

Bīrūnī's statement is a reflection of the fact that in the Ptolemaic lunar model ([20], pp.192-194) the distance from the earth to the epicycle center is not a function of $\bar{\lambda}$, but of the mean elongation, η say, the difference between the lunar and solar mean longitudes. The double elongation is 2η. The epicycle center will be at deferent apogee when $2\eta = 360^\circ$ (mean opposition), and when $2\eta = 720^\circ$ (mean conjunction). It will be at deferent perigee when $2\eta = 180^\circ$ and 270° (mean quadrature).

For a discussion of the lunar sectors see [13], p.252.

63:18 - 64:6. Here Bīrūnī apparently reconciles himself to an approximate determination of the epicycle sectors for the Ptolemaic planetary models. The method he advances is not clear to us but evidently it is an effort to make the deferent eccentricity modify the epicycle sector boundaries. The use of half the equation of the center for the sectors reckoned according to the "first opinion" (mean distance) may be a consequence of the fact that half the maximum epicyclic equation (or half its sine) is used in determining the boundaries of the epicycle distance sectors. (See the comment to 41:2.) For the "other opinion" i.e. for sectors reckoned according to mean angular velocity, the entire equation of the center is used in modifying the epicycle sector boundaries.

64:7. This individual is otherwise completely unknown to us.

64:12-17. Bīrūnī is, of course, right. The arguments of

the mean and anomalistic motions are independent variables, and to assume that the moon is in conjunction is to assume nothing whatsoever about the value of the anomaly.

<u>64:19 - 65:2.</u> This approximation to the anomalistic month, $27^{d}\ 13;20^{h} = 27;33,20^{d}$ is a crude Babylonian parameter.

<u>65:7-18.</u> Here Bīrūnī resorts to sarcasm, as was not uncommonly his wont. In 65:11 we have restored احر to اخذ.

The sentence beginning in 65:18 is unintelligible to us. The word البحاين occurs also in 61:12 where we have translated it as "tides", perhaps with no good reason.

<u>65:19 - 66:8.</u> Here Bīrūnī concocts a way of making sectors I and IV for the sun smaller than II and III, as stated in 65:6. If, for whatever reason, the deferent is divided into quadrants as shown in Figure 12, then the quadrant AB will subtend less than a right angle at H. That is, when $\bar{\lambda}=90°$, $\lambda < 90°$. If now the sectors are regarded as arcs of the "parecliptic" (not shown in the figure, cf. the comment to 18:12) rather than of the deferent, sectors I and IV will be exceeded by II and III. In 66:6 and 66:7 we again restore مرتبة in the text to مرئية.

<u>66:9 - 67:13.</u> This is the opening passage of a considerable section, which reaches to 71:11, and which describes the physical attributes implied by the presence of a celestial body in a particular sector. The author deals first with deferent distance sectors, stating that insofar as brightness is concerned, the object will be at a minimum at apogee, maximum at perigee, and will be increasing in brightness (since it is approaching the earth) in sectors I and II, and decreasing in III and IV. This is based on the assumption that brightness varies inversely with distance from the earth; it ignores the phases of the planets, and it neglects the effect of the epicycle. Bīrūnī likens this to the waxing and waning of the moon, pointing out that in the case of the latter it is not a matter of the apparent

diameter, but of the portion of the lighted face which is turned toward the earth.

<u>67:14 - 68:12.</u> Turning next to the velocity sectors the author observes that angular velocity is a minimum at the apogee, maximum at perigee, and monotonic in between in the sense that it is always increasing in sectors I and II and decreasing in III and IV.

In 68:1 we restore الثاني of the text to الثالث.

Bīrūnī states (68:5) that the equation is increasing in odd-numbered sectors and decreasing in the others, and that the statement applies both to deferent and to epicycle sectors. We would say under the same circumstances that the equation was increasing or decreasing in absolute value.

For the deferent (but not the epicycle) sectors the "computation is diminished" (68:6), meaning, as we would put it, that the correction to be added to the mean longitude is negative in sectors I and II. In the other two it is positive.

Bīrūnī seems to share our uncertainty as to the meaning of "number" (<u>al-ᶜadad</u>) in 68:9. In tabulating a function having a period of 360° and symmetrical about 180° it was customary to enter the arguments in a pair of adjoining columns, each known as "the column of the number" (<u>saṭr al-ᶜadad</u>) from 0° to 180° down one, thence back up the other to 360°. This may be what is meant in 68:10. Many zījes have tables of functions giving reciprocals of the distances from the center of the universe to sets of points on the deferent or epicycle. Perhaps this is the meaning of 68:11.

<u>68:13 - 69:8.</u> Since the rotation in the lunar epicycle is contrary to the rotation in the deferent, the effects of the former resemble qualitatively the solar motion. For if the sun's eccentric deferent were replaced by a suitable epicycle, rotation in it would likewise be opposite to that of the mean motion. In both cases when the object is in sectors I and IV the epicyclic motion tends to slow down

the motion with respect to the center of the universe, and conversely to speed it up in the remaining two sectors.

69:9 - 71:2. The planetary models differ from that of the moon in two respects. For one thing the rotation in the epicycle is in the same direction as that around the deferent, hence the epicyclic motion accelerates the latter in sectors I and IV and decelerates it in the other sectors. But further, the deceleration for all the planets is so drastic as to cause each to become retrograde for a while in the vicinity of the epicyclic perigee. In this passage, Bīrūnī recites the sequence of events as the planet passes through the epicyclic sectors in succession. In 70:2 we restore الباى of the text to الباري.

He mentions a second time (70:4, cf. 16:1 above) the primitive explanation of the planetary retrogradations as being caused by halters, which here he speaks of as extending from each planet to the sun. Thus stated, the notion has much to recommend it, and we too can think of the retrogradations as consequences of a restraint exerted from the sun - the pull of its gravitational field.

Since Bīrūnī considers the absolute value of the equation (cf. the comment to 68:5 above), his statement here (70:17) to the effect that the epicyclic equations are increasing in the first and third sectors and decreasing in the others is valid.

With the "computation" (cf. 68:6 above, and the comment), on the other hand, the sign of the equation is taken into consideration. Now the epicyclic and deferent effects are opposite, since in the first and second epicycle sectors the equation is additive, and subtractive in the third and fourth.

As with the deferent sectors, the apparent diameter of the object is less in sectors I and IV, and greater in II and III.

71:2-5. In referring here to increases and decreases of latitude Bīrūnī utilizes the concept he has previously

developed (see 8:10 above and the comment) of north as connoting "up". The "inclined heaven" (al-falak al-mā'il) is the plane of an orbit which is slightly inclined with respect to the ecliptic. The plane of the inclined heaven intersects the celestial sphere in a great circle which in turn intersects the ecliptic in a pair of points, the ascending and descending nodes. Apparently the latitude sectors are the four quadrants of this great circle which commence with the ascending node and proceed from west to east overhead. The latitude will be increasing (in absolute value) in sectors I and III and decreasing in the other two. It will be "ascending" (i.e. proceeding northward) in I and IV and "descending" in the other two.

71:5-11. In like manner the ecliptic is divisible into four sectors by the four astrological centers (see the comment to 7:4 above). Their manner of numbering is described clearly by the author.

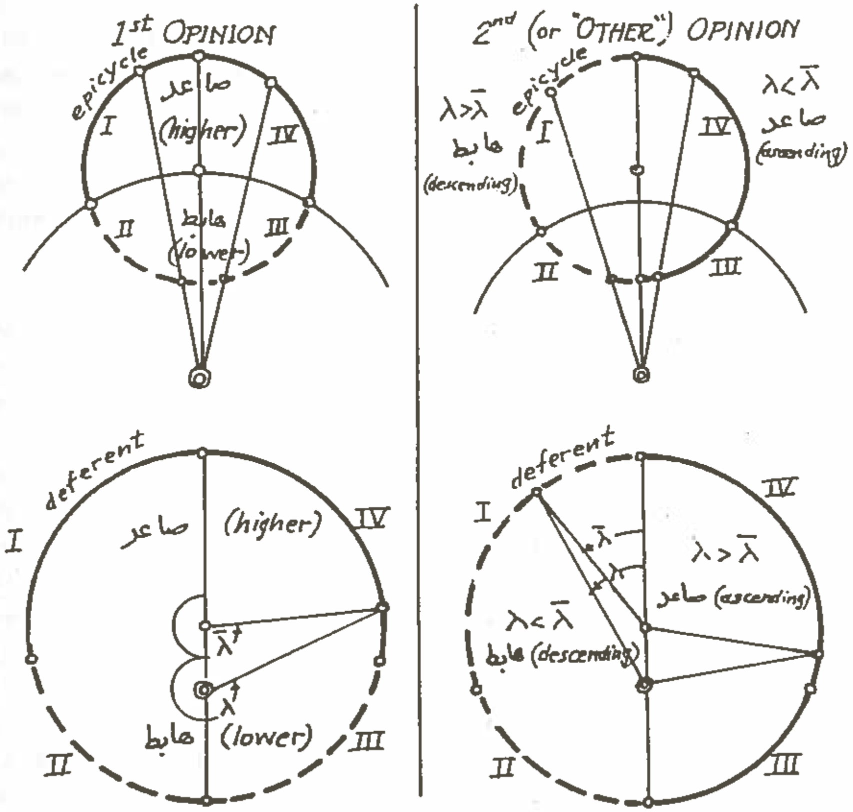
The results of this whole section on "increases and decreases" (66:9-71:11) are displayed in the table which appears on page 104 of the printed text (our page 116).

71:12 - 73:3. Here begins the detailed discussion of the transit in thickness, a topic to which the consideration of sectors has led, and which will take up most of the remainder of the book.

The planets were thought of as being confined, each within one of a set of hollow, concentric spherical shells. Since the shell of Venus, say, was inside that of Mars, and separated from it by intervening shells, the radius vector of the former could never equal, much less exceed, that of the latter. Hence Venus could never actually transit, i.e. cross over, Mars in the sense of thickness. It was customary instead to compare the radius vectors of two planets relatively, each with respect to its own maximum and minimum distances (71:15). The situation is complicated by the fact that the length of the radius vector, like the true longitude, is a function of two variables,

the mean longitude and the anomaly. To put the same statement differently, the location of the planet with respect to the sectors, both of the deferent and the epicycle, is relevant.

The rule given in 71:19 ignores these difficulties by saying that if $\bar{\lambda} < \lambda$ the planet is ascending, i.e. receding from the earth; if $\bar{\lambda} > \lambda$ it is descending. These conclusions are valid for a planet having an epicycle alone; the effect of the eccentric deferent is just the opposite. Since for most planets the eccentricity is small, perhaps it was neglected, or this may be an additional piece of evidence pointing to the theory that the original "users of the transit" were operating with a pre-Ptolemaic variety



of astronomy in which there was only one equation per planet. This seems to be the case in the discussion of 92:3-7 below.

In passage 72:14-17 is the first juxtaposition of the two conflicting "opinions" regarding ascent and descent. According to the "first opinion", encountered already in 15:10 and 59:9 above, a planet is <u>ṣāʿid</u> (here "higher") whenever the length of its radius vector exceeds its mean value. This will occur when it is in (distance) sectors I and IV. In sectors II and III it is <u>hābiṭ</u> (here "lower").

According to the "second opinion" the planet is <u>ṣāʿid</u>, "ascending", when the length of the radius vector is increasing, and this is the case in sectors III and IV. When the vector's length is decreasing the planet is <u>hābiṭ</u>, "descending", which is the case in sectors I and II.

The situation is displayed graphically in the figure on page 168 for both the epicycle and eccentric hypotheses. In this connection, see 92:10 below.

<u>73:4-8.</u> This passage makes little sense to us. If it could be read "the mean of each (inferior planet) is the sum of the sun's mean and its (the sun's) equation", all would be well, but the text is unequivocal.

As for 73:5, the difference between the mean longitudes of the sun and a <u>superior</u> planet is indeed the planet's mean anomaly, but the reference here seems to be to the <u>inferior</u> planets.

<u>73:9-74:1.</u> Here Bīrūnī voices precisely the criticism which we have made in the comment to 71:12 above.

<u>74:2 - 77:2.</u> In order somehow to take both equations into consideration, Abū Maʿshar (see 5:10 above) adopted the following expedient. He formed two numbers: ① a constant, the arithmetic mean of the two maximum equations, and, ② a variable, the algebraic sum of the values of the two equations at the instant in question. If ② > ① the planet

is ascending; if ② < ① it is descending, and when ② = ① it is at mean distance. This arrangement is so patently absurd that for Bīrūnī to marshal a page of text and the elaborate Figure 13 to refute it seems, as the saying goes, like sending a man to do a boy's job. Be that as it may, several examples are exhibited in which the rule fails. For instance, when the planet is at M as shown (75:14) its epicyclic equation is zero and its equation of the center half the maximum, or less. Clearly now ② < ①, whence, according to the rule, it is "descending", i.e., at distance less than the mean. In point of fact it is at a distance greater than the mean.

In 74:11 we restore the لنمو of the text to لتمرّ; in 76:8 التمريلين of the text is restored to التعريلين.

<u>77:3 - 78:15.</u> Al-Khāzin ([14], p.137) was a fairly well-known scientist of Khurāsān who flourished in the middle of the tenth century. His criticism of Abū Maʿshar and Bīrūnī's criticism of him are equally obscure to us.

<u>78:16-80:7.</u> As remarked in connection with 72:18 above, it was believed that any given planet exerted an influence on planets in aspect with it by casting rays, missiles as it were, upon them. This passage indicates that it was customary to modify the point of incidence of the ray depending on the casting planet's distance from the earth. The application of the doctrine, however, was not uniform. In 79:1, for instance, it is indicated that when the planet is at a distance greater than the mean the ray is shortened. That is, the place it strikes the ecliptic is nearer the position of the casting planet than if the latter were at mean distance. When the earth-planet distance is less than the mean the effect is the opposite, the ray is lengthened.

On the other hand, in 79:19 if $\lambda > \bar{\lambda}$, which says nothing about the earth-planet distance, a ray cast forward is to be lengthened, while one projected backward is to be shortened.

In 80:5 is the first mention of the Jewish astrologer Messehalla, known in Arabic as Māshāllāh (more properly transliterated <u>Mā shā' Allāh</u>) ibn Atharī al-Basrī, who flourished at the eighth century Abbasid court in Baghdād. His fame was second only to that of Abū Maʿshar. The son mentioned in 63:7 above is otherwise unknown to us. See [26], p.5.

<u>80:8 - 85:17.</u> <u>Magnitude of Transit</u>.

In this passage Bīrūnī describes variant methods of computing the "magnitude of transit" (<u>miqdār al-mamarr</u>), a concept introduced in 79:13 above. It is determined by taking the difference between the planet's mean and true longitudes and multiplying it by a constant, k.

In 80:11 the constant is determined by putting

$$k = \frac{1}{6\frac{1}{4}} = \frac{4}{25}.$$

Abū Maʿshar's predecessors (80:12) obtained the same thing by setting

$$k = 8/50.$$

The rule (80:16) common to Māshāllāh (cf. 80:5 above), the Shāh Zīj (cf. 24:7), and al-Jawzaharī is

$$k = \frac{800}{3600} \cdot \frac{360}{500} = \frac{2}{9} \cdot \frac{18}{25}.$$

The zīj of the latter individual is otherwise completely unknown to us. Deletion of a single letter in his name would make it al-Jawharī (fl.830) one of the astronomers of the caliph al-Ma'mūn, and to whom a zīj is attributed ([14], p.136.)

The practise of Abū Maʿshar (cf. 5:10 above), is to put, for the planets (81:13),

$$k = \frac{2}{9} \cdot \frac{36}{50}.$$

For the sun and the moon, however, he put (81:17)

$$k = \frac{4}{9} \cdot \frac{36}{50},$$

for reasons which neither we nor Bīrūnī can explain.

According to some "books of the astrologers" (82:14)

$$\frac{40}{180} \cdot \frac{18}{25} \quad \left(\frac{4}{25}\right),$$

although in some cases (83:1)

$$\frac{216'}{5'} \quad \left(\neq \frac{216'}{5} = \frac{3;36}{5} = \frac{3\frac{3}{5}}{5} = \frac{18}{25}\right)$$

was erroneously put for the second ratio, 18/25.

Abū ʿAlī (83:5) is unknown to us. He garbled the above rule to

$$k = \frac{40}{100} \cdot \frac{18}{25} \,,$$

while Māshāllāh's books sometimes had (83:7) successive corruptions of

$$\frac{40}{180} \rightarrow \frac{160}{180} \rightarrow \frac{60}{88} = \frac{15}{22} \,.$$

Al-Farghānī was also one of the astronomers of al-Ma'mūn ([26], p.186). The passage concerning his rule (83:15) depends on the fact that

$$k = \frac{4}{25} = 0;9,36 = 9\frac{3'}{5} = 576'' = (48)(2)(6)''$$

expressed in sexagesimals. It looks as though the division by five (83:16) is redundant. Concerning "elevation", see the comment to 26:4 above.

In 83:15 we restore الفصل to الفضل, "excess" or "difference" between the mean and true longitudes. If this difference is in minutes, multiplying by (42)(2)(6) will indeed give the result in thirds (minutes times seconds) as 83:17 says.

Concerning Ibn al-Bāzyār (84:15), see the comment to 10:17 above.

For Ḥabash (cf. the comment to 22:13) the rule is (84:17),

$$k = \frac{7}{22} \quad \left(\approx \frac{1}{\pi}\right) \,.$$

In the Berlin version of this zīj (15 in [14], f.114v) there appears a short section on the transit. In it the rule says to multiply the equation of the center by 7/12. Probably the 12 (يب) is the result of some scribe having

left off the frontal stroke of the initial kāf in 22 (كب). Precisely this error is noted (85:7) in the Kāfī Zīj, a work otherwise unknown to us.

The proverbial expression in 85:6 has a play on words between aghrab, "more strange", and ghurāb, "crow".

The table mentioned in 85:9 is probably just a multiplication table giving in sexagesimals

$$kn = \frac{4}{25} n = 0;96,36 \; n, \qquad n = 1,2,3,\ldots,59.$$

This would be a convenience in computing magnitudes of transit.

Having looked at all the variant methods of what is essentially the same operation, it is well to ask the reason for performing it. A clue is given by 79:13, which says that magnitude of transit is the amount the planet rises or descends, presumably away from or toward the earth. If we restrict consideration to a planet in a single epicycle sector, say I, then the difference between its mean and true longitudes is indeed a (non-linear) measure of its descent from its apsidal position of maximum distance. Once the planet has passed into sector II, its equation is a measure of its ascent from the perigee position it is then approaching, and so on.

This does not explain the coefficient k, and for it we have in fact no satisfactory explanation. The most fruitful suggestion has been made by O. Neugebauer, who remarks that $3\frac{1}{8}$ is an attested Babylonian approximation to the number π. (Cf. [20], p.47.) Using this, our formula for magnitude of transit becomes

$$ke = \frac{e}{2\pi} \,.$$

As we have seen frequently (e.g. in 32:1 ff.) it was customary in Hindu astronomy to fix the sizes of epicycles by giving their circumferences in degrees, where a degree stands for the 360th part of a planet's mean orbit. The inventor of the transit in thickness concept may have sought to transform his circumferential e into a radial

distance by dividing by 2π. This conjecture is reinforced by the use of a better approximation to π in the zīj of Ḥabash and in some copies of the Shāh Zīj as noted above.

85:18 - 89:8. The Doctrine of Abū Maʿshar

Except for the final section the remainder of the book is given over to a detailed discussion of the usage of two astrologers, or groups of astrologers. Abū Maʿshar is the first.

He computed a set of constants

$$w_{max} = \frac{4}{25} e_{max}$$

for each planet and for both varieties of equation, the epicyclic and the deferent. The w_{max} is called the "apogee chord" if the e_{max} is that of the deferent; the "radius chord" is obtained from the epicyclic e_{max}.

For a given instant form also

$$w = \frac{4}{25} e,$$

known as the "partial chord" (al-watar al-juz'ī) of the apogee or radius depending on which equation was used. Notice that the two equations are treated separately.

Now

$$\mu = \frac{w}{w_{max}}$$

is called the "minutes of transit" (daqā'iq al-mamarr). Obviously one would obtain the same result by forming e/e_{max}. This is pointed out by Bīrūnī (86:17).

Assuming that the sector is known, the determination of μ gives us a measure of the planet's elevation or depression with respect to its mean distance. If a planet happened to be in the third sector $\mu = 0$ would indicate that it was at the initial point of the sector, at minimum distance from the earth. If $\mu = 1$ it would be (approximately) at mean distance, at the endpoint of this sector. Any intermediate μ would indicate an interior point of the sector, and one μ larger than another would insure that the planet in the first case is farther from perigee than in the second. An essential point is that division by w_{max}

insures that all epicycles and eccentricities for different planets are made comparable, since all are cut down, as it were, to a standard size.

Bīrūnī gives a table of w_{max} for all the planets, presumably from Abū Maʿshar's zīj. In the printed text this table has somehow had its place exchanged with another, and appears on page 91. We have restored it to its proper place on page 87. In all the entries we have restored the text's وثر to وتر.

In principle, since each entry in the table (except for the sun and moon) is of the form

$$\frac{4}{25} e_{max} = 0;9,36\ e_{max} = \frac{1}{6;15} e_{max},$$

division of each by 0;9,36, or multiplication by 6;15, should present us with a complete set of Abū Maʿshar's parameters. In fact the text, in addition to obvious misreadings, is rather corrupt. In some cases we will find it necessary to work both ways in order to obtain probable restorations. In other cases no restoration seems feasible.

For the two luminaries we recall Bīrūnī's statement in 81:16 that the coefficient is double the usual value. The standard solar e_{max} is 2;14°, and we notice that

$$2;14(2)(0;9,36) = 0;42,52,48 .$$

This is not identical with the table entry, but trial of 2;13, the only other probable value, yields a result so different from the text that 2;14 seems clearly the proper one. The same goes for the apogee chord of Venus.

For the moon,

$$4;56(2)(0;9,36) = 1;34,43,12 ,$$

to which the text is easily restorable.

For the apogee chord of Saturn,

$$8;37(0;9,36) = 1;22,43,12 ,$$

also close to the text.

For the radius chord of Saturn,

$$5;44(0;9,36) = 0;55,2,24 ,$$

which is irreconcilable with the text.

For the apogee chord of Jupiter,

$$5;6(0;9,36) = 0;48,57,36 .$$

which entails the restoration of one digit in the text.

For the radius chord of Jupiter,

$$10;52(0;9,36) = 1;43,19,12 \ ,$$

which involves the easy restoration of two digits.

Operating in the reverse direction for the apogee chord of Mars, we obtain

$$1;47,12,0(6;15) = 11;10,0,0,0 \ ,$$

which is the parameter ascribed to al-Fazārī in 31:8 above.

For the radius chord of Mars,

$$40;30(0;9,36) = 6;28,48,0 \ ,$$

which requires only the easy restoration of the text's ك to ع.

The apogee chord of Venus has been disposed of above. For the radius chord,

$$47;11(0;9,36) = 7;32,57,36 \ .$$

The restoration of the text to this is less drastic in the abjad sexagesimals than would appear from the transcription.

For Mercury, maximum equations of 4;0° and 21;30° give complete correspondence with the text.

The results of these investigations are displayed in the first column of the table below.

	Abū Maᶜshar	Māshāllāh
☉	2;14°	2;13°
☾	4;56	4;56
♄ center	8;37	8;37
♄ anomaly	–	5;44
♃	5;6 10;52	5;6 10;52
♂	11;10 40;30	– 40;31
♀	2;14 47;11	2;13 47;11
☿	4;0 21;30	4;0 21;30

In 88:17 we restore the ابعاض of the text to ابعاد.

Evidently Bīrūnī's "due time" (89:8) was sufficiently delayed to carry out his hope, for in the bibliography of his own works made late in his life he lists a treatise on the shortcomings of Abū Maᶜshar's zīj. Unfortunately it has not come down to us.

89:9 – 103:14. The Doctrine of ᶜUmar ibn al-Farrukhān and Māshāllāh

Ibn al-Farrukhān (fl. 770) was one of the early Muslim astronomers who translated scientific works from Persian into Arabic ([26], p.7). The quotation from a work of his given in our text is clear enough, and we will discuss presently the technique it describes. Of much more interest is the table referred to in 90:10. The table itself has been misplaced in the printed text and appears on page 87 where Abū Maᶜshar's table of chords should be. In the translation we have restored both to their proper positions.

The word منيرين in 90:10 has been restored to منبرين. In the table الوتد has been restored to الوتر, and لجريه to تجزية.

The table consists of what we will call "apportioning coefficients", ratios between all pairs of maximum equations, except that deferent equations are paired with deferent equations, epicyclic with epicyclic. In the upper triangular array of the table the sun and moon do not appear, since in this system the moon, like the sun, has only one inequality in its motion. The ratios are so chosen as to be greater than or equal to one, and are displayed in sexagesimal seconds converted into decimal integers. For example, the tabular entry giving the ratio between the maximum deferent equations of Saturn and the sun is 13994, meaning

$$3;53,14 = 3(60^2)+53(60)+14.$$

The existence of this table implies that if any one epicyclic or deferent maximum equation is known all the others may be computed in terms of it. Moreover, since there are many ratios involving any particular planet, results can be checked several ways and scribal errors restored with complete certainty. Initial values for beginning the process are at hand, in the passage of the text where parameters of the Shāh Zīj are cited. Thus it has been possible to obtain all of the maximum equations used by Māshāllāh's version of the Shāh Zīj except for the

deferent equation of Mars, since all the ratios involving it are mutually inconsistent. The results are shown in the second column of the table in the commentary to 85:18 above.

The "one of the (above-)mentioned opinions" (90:13) followed by ʿUmar is the "second opinion" described in the comment to 71:12. Here Bīrūnī states again (90:15) that to infer ascent or descent it is insufficient simply to compute the difference between the mean and true longitudes, for the equation thus found is compounded of the epicyclic and eccentric effects, which may appear in any arbitrary combination. There follows a long discussion based on Figure 16. The circle ABJD is taken to represent either a deferent or an epicycle. Bīrūnī's statement at 92:1 is a consequence of the fact that in the Hindu planetary theory the equation of the center is computed by the "method of sines". (Cf. the comment to 24:19, also [21], p. 177). Hence maximum equation occurs at the quadrants.

In reading 92:3-11 the reader may find it useful to refer to the figure accompanying the comment to 71:12.

Bīrūnī proceeds (93:12-93:17) to compare the situations of two planets, Y and Z, located on the same orbit. His remarks about projecting them on the line of apsides AJ make little sense unless he is regarding the circle as an epicycle. Moreover, since both are on the same epicycle, some method must have been in use for expanding the smaller epicycle to the size of the larger. This furnished a clue to the application of the table of apportioning coefficients which follows 90:11. For these coefficients may be regarded as ratios between epicycle radii, or between maximum equations. Before comparing elevations between planets having different epicycle radii, the elevation of the planet of smaller radius should be multiplied by the proper apportioning coefficient.

In the final passage (93:18 - 95:9) of the section, consideration is given to the ascents and descents of planets in different sectors. Again the tacit assumption seems to be that the circle of the figure is an epicycle.

The book of Māshāllāh mentioned in 96:4 may have survived in Latin translation. (See [26], p.6).

Concerning the "first" and "second" opinions, see the comment to 71:12 above.

The two quotations from Māshāllāh (97:12-18) are indeed conflicting. The first is the usual "second opinion" In the second quotation the quantity, four and a half signs, makes sense, for the magnitude of Venus' first velocity sector is close to 135° ([13], p.250) which is just four and a half zodiacal signs. But to take sectors I and IV as falling and II and III as ascending is just the opposite of the "first opinion". As Bīrūnī sarcastically says, this is a "third opinion".

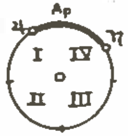
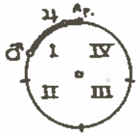
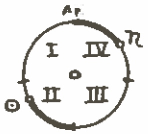
He now proceeds to examine a worked example of a transit computation performed by Māshāllāh. The text gives a horoscope, cast for the taḥwīl, the instant of vernal equinox for the year in which a certain Jupiter-Saturn conjunction occurred. This particular year was of special astrological interest as being one of a "shift of the transit" (explained in the comment to 6:19 above). For such considerations each of the four triplicities was associated with one of the four classical elements. The earth triplicity included Taurus, Virgo, and Capricorn; the air triplicity included Gemini, Libra, and Aquarius.

The horoscope (the ascendant) had a longitude of 140°, and the longitudes and apogees of the four planets dealt with are shown in the table below as given in the text. O. Neugebauer dates the configuration as being that of 20 March 333. For comparison we show also the apogees of the same planets as given in the Sūryasiddhānta known to Varāhamihira and the Khaṇḍakhādyaka ([9], p.xii).

	The Horoscope		
		Apogees	
	λ	Shāh Zīj	Old S.-S. and Khand.
♄	189;8°	240°	240°
♃	172;44	160	160
♂	344	115	110
☉	0	80	80

In 99:16 we restore an <u>alif</u> to وجه to obtain أوجه.

Māshāllāh had evidently picked the three pairs of planets associated in the table below this paragraph, why these three we do not know, and for each he computed the elevation of the "upper" planet over the "lower".

	True longitudes of conjunction	Equations of the center	Division by the apportionment
♃ over ♄	 189;8° 189;8°	148′ 400′ ♃+♄ = 548′	$\frac{548}{(♄/♃)} = \frac{9;8}{1;41} \approx 5;28°$
♃ over ♂	 172;44° 172;44°	67′ 566′ ♃+♂ = 630′	$\frac{630}{(♂/♃)} = \frac{10;30}{2;11} \approx 4;48°$
♄ over ☉	 189;8° 189;8°	400′ 125′ ♄−☉ = 275′	$\frac{275}{(♄/☉)} = \frac{4;35}{3;53} \approx 1;10°$
			$\Sigma = 11;26°$ $11^{y}\,5^{m}\,6^{d}$

In order to verify his results, Bīrūnī attempts to recompute the component equations, working backward from the true longitudes, and apparently with a copy of the Shāh Zīj at hand. He begins with Saturn, alleged by Māshāllāh to have $\Delta\lambda = 400' = 6;40$ descending (99:16, according to the "second opinion"). In the following we will use $\Delta\lambda$ for the difference between the true and mean longitudes, the customary planetary symbols as subscripts, as well as the subscripts a, c, and Ap for anomaly, center, and apogee respectively.

To obtain the approximate argument of the center, put

$$\lambda_{♄} - \lambda_{Ap,♄} = 189;8° - 240° = 309;8°.$$

The text (99:19) has 309°, whence, says Bīrūnī, from the Shāh Zīj one obtains

$$e_c(309°) = 6;35°.$$

By way of comparison, the published version of al-Khwārizmī's zīj ([16], p.139) has

$$e_c(309°) = 6;37°.$$

Both are close to the 6;40 given by Māshāllāh, but the location of the mean planet is also affected by the size of the anomalistic equation. Bīrūnī therefore attempts to approximate the argument of the anomaly as being (100:5)

$$\lambda_{☉} - \lambda_{♄} = 360° - 189;8° = 170;52°.$$

The corresponding tabular entry is

$$e_a(170;52°) = 1;8°,$$

to which may be compared nearby entries in al-Khwārizmī (p.143) of

$$e_a(170°) = 1;6°,$$

and

$$e_a(171°) = 0;55°.$$

Then

$$\lambda_{♄} - e_a = 189;8° - 1;8° = 188°,$$

the <u>adjusted center</u> (100:7). From this an improved argument of the center is

$$\lambda_{c,♄} - \lambda_{Ap,♄} = 188° - 240° = 308°,$$

and the corresponding equation (100:9)

$$e_c(308°) = 6;41°.$$

Al-Khwārizmī's corresponding entry is

$$e_c(308°) = 6;43°.$$

An improved value for the mean longitude of Saturn is (100:11)

$$\bar{\lambda}_{♄} = 188° - 6;41° = 181;19°.$$

whence $\Delta\lambda = 189;8^\circ - 181;19^\circ = 7;49^\circ = 469'$.

Of this Bīrūnī now computes the magnitude of transit (100:13) by putting (cf. 80:10)

$$\frac{4}{25}(\Delta\lambda) = \frac{4}{25}(469') \approx 75'.$$

With this Bīrūnī drops Saturn and takes up Jupiter. He rightly criticizes Māshāllāh's putting the longitude of Jupiter equal to the equinoctial longitude of Saturn, saying that by the time the conjunction occurs the latter will have moved somewhat. Be that as it may, this gives for the approximate argument of the center (101:1)

$$\Delta\lambda_{♃} = \lambda_{♃} - \lambda_{Ap,♃} = \lambda_{♄} - \lambda_{Ap,♃} = 189;8^\circ - 160^\circ = 29;8^\circ.$$

From the Shāh Zīj

$$e_c(29;8^\circ) = 2;28^\circ = 148',$$

which is the same as the corresponding entry for al-Khwārizmī ([16], p.144)

$$e_c(29^\circ) = 2;28^\circ.$$

Without this time giving the details of the determination, Bīrūnī announces as an improved value, $e_c = 2;19^\circ$. Neither this, however, nor the magnitude of transit computed above is utilized. He reverts to the 148' of 101;2 and the 400' of 99:16, adding them to get 548' = 9;8 in 101:6. This is divided by the appropriate apportioning coefficient from the table following 90:11 (cf. 90:4),

$$\frac{548'}{6082} = \frac{9;8(5;6)}{8;37} \approx 5;24.$$

The text (101:7) has 5;28, which value is confirmed later. This is the elevation of Jupiter over Saturn.

Now the Jupiter-Mars pair is considered. Bīrūnī starts with Jupiter, which is alleged to be $67' = 1;7^\circ$ above the sector (101:9). The approximate argument of the center is

$$\lambda_{♃} - \lambda_{Ap,♃} = 172;44^\circ - 160^\circ = 12;44^\circ,$$

although the text has 12;45. According to 101:11,

$$e_c(12;45^\circ) = 1;9^\circ,$$

which is close to al-Khwārizmī's (p.144)

$$e_c(13^\circ) = 1;8^\circ.$$

By separating out the two equations Bīrūnī obtains (101:12) $e_c = 1;11^\circ$. Again the first approximation is closer to Māshāllāh's value than is the improved version.

For Mars the claim is 566' = 9;26 below the sector (101:14). Put $\lambda_{♂} = \lambda_{♃} = 172;44^\circ$. Then the approximate argument of the center will be

$$\lambda_{♂} - \lambda_{Ap,♂} = 172;44^\circ - 115^\circ = 57;44^\circ,$$

although Bīrūnī gives (101:16) 57;45°. Now

$$e_c(57;45^\circ) = 9;27^\circ,$$

the same as the entry of al-Khwārizmī

$$e_c(58^\circ) = 9;27^\circ.$$

Upon adding the minutes for both planets the text reports 630', although 67' + 566' = 633'. It would appear to us that since the sectors are the same the numbers should have been subtracted, and in this Bīrūnī (102:4) seems to concur.

Nevertheless the result is divided by the proper apportioning coefficient from the table following 90:11 to obtain (102:2)

$$\frac{633'}{7896''} = \frac{10;33(5;6)}{11;10} \approx 4;48,$$

a result which is confirmed later in the text; it is the elevation of Jupiter over Mars.

In 102:4 we restore the text's التقاصيل to التفاضيل.

To account for Māshāllāh's having added the minutes Bīrūnī conjectures that perhaps he took Mars as being in its actual position, $\lambda = 344^\circ$, since then, insofar as the apogee is concerned, it would be ascending. The approximate argument of the equation would then be

$$\lambda_{♂} - \lambda_{Ap,♂} = 344^\circ - 115^\circ = 229^\circ.$$

The text states (102:8) that then

$$e_c(229^\circ) = 3;15^\circ,$$

which is widely divergent from al-Khwārizmī's corresponding entry (p.154) of 8;24°. Perhaps the text should be restored to [8];15°.

Bīrūnī also tries putting Mars in opposition to Jupiter, at $\lambda_{♂} = 352;44^\circ$. Then the argument of the equation would be

$$\lambda_{♂} - \lambda_{Ap,♂} = 352;44^\circ - 115^\circ = 237;44^\circ,$$

whence

$$e_c(237;44^\circ) = 7;30^\circ,$$

whereas the corresponding entry in al-Khwārizmī's table (p.154) is 9;27°. It would be plausible to restore the سبع (seven) of the text to تسع (nine), yielding [9];30° .

Finally Bīrūnī tries putting Mars at the point in opposition to its given position. The approximate argument of the center becomes

$$\lambda_{♂} - \lambda_{Ap,♂} = 164^\circ - 115^\circ = 49^\circ,$$

and the equation (102:11)

$$e_c(49^\circ) = 7;1^\circ.$$

Al-Khwārizmī's entry (p.151) is again 8;24°, by virtue of the symmetry of the equation function, since 229°-49°=180°.

The text now passes on to the Saturn-Sun couple.

In 102:14 the word ثلاثون must be restored to ثمانون.

Put $\lambda_\odot = \lambda_{♄} = 189;8^\circ$, whereupon the approximate argument of the solar anomaly becomes

$$\lambda_\odot - \lambda_{Ap,\odot} = 189;8^\circ - 80^\circ = 109;8^\circ.$$

Although the descent Māshāllāh actually takes is 125' = 2;5°, Bīrūnī says (102:17) the tabular entry opposite the argument is

$$e_\odot(109;8^\circ) = 2;6^\circ.$$

Al-Khwārizmī (p.135) has

$$e_\odot(109^\circ) = 2;6,18^\circ.$$

Had Māshāllāh taken the sun in its proper position

(the vernal point), or opposite it (the autumnal point), the argument of the anomaly would be 80°, giving (102:19) for both assumptions

$$e_\odot(80^\circ) = e_\odot(80^\circ + 180^\circ) = 2;10^\circ.$$

Al-Khwārizmī's corresponding entry (p. 134) is 2;11,44°.

For Saturn a value of 400' has previously been computed (99;16). That of the sun being in the same direction we subtract:

$$400' - 125' = 275' = 4;35.$$

Now divide (103:4) by the proper apportioning coefficient from 90:11,

$$\frac{275'}{13994''} = \frac{4;35(2;13)}{8;37} \approx 1;10,$$

which is the elevation of Saturn above the sun.

This added to the other two "elevations", from 101:7 and 102:2, gives

$$5;28 + 4;48 + 1;10 = 11;26^\circ.$$

This is converted into time, $11^y\ 5^m\ 6^d$, by the engaging expedient of putting twelve months equal to a year, and thirty days equal to a month.

We have consolidated all these operations in the table on page 180. In the three circular diagrams we have superposed the zodiacal positions of the associated pairs of planets in such fashion as to make their apogees coincide, thus forcing the sets of sectors into a standard position. With the aid of these diagrams we note that the three planets "over" their associates are correctly placed in the sense that each of the three is closer to the apogee than its mate. But this is the best that can be said for Māshāllāh. Bīrūnī is right in claiming that the 566' and 67' of this second pair should be subtracted rather than added.

The division by the apportionment makes no sense either. If in the first couple, say, Saturn's descent of 400' had been divided by the apportioning coefficient of 1;41, this would have made the descents of the two planets

comparable, compensating for Saturn's greater eccentricity.

Whatever Māshāllāh's shortcomings from a logical point of view, it has been possible to put this second-hand fragment of his labors to work, and to show that the planetary equations of the center in the zīj he used, the Shāh, were computed by the "method of sines" ([13], p.259).

103:15 - 104:1. The table referred to here, and which takes up most of page 104 of the text is a consolidation of the conclusions reached in 66:9 - 71:11.

104:1 - 106:2. This discussion, based on Figure 18, is a clear description of how to compute the earth-planet distance for a given instant and for the Ptolemaic model. This is to be done in terms of the deferent radius, the planet's mean distance. Two planets' distance may then be compared, although no allowance is made for the fact that even with this arrangement the maximum distances of any two planets will differ. Far be it from us, however, to offer an additional scheme.

106:3 - 107:3. This is a final reversion to the transit in latitude, first discussed in 8:9-11:17. Bīrūnī seems to be saying that for it, as in the preceding passage, the maximum variations are not to be "normed" to a common unit, rather the latitudes of the two planets are to be compared as they are.

107:4-7. This is a closing reference to the notion of elevation with respect to the horizon. See 7:8 and the commentary. The "azmān" there referred to are units of time obtained by putting the 360° of daily rotation equal to twenty-four hours. Thus an hour equals fifteen "times".

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References to the translation give page and line of the text, separated by a colon. The corresponding numbers appear along the left-hand edge of each page of the translation. Numbers preceded by a double asterisk (**) are references to pages of the commentary.